

Optimal Irrigation Scheduling

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ABSTRACT

Abstract of a thesis submitted in partial fulfilment of the
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An optimal stochastic multi-crop irrigation scheduling algorithm was developed which was able to incorporate complex farm system models, and constraints on daily and seasonal water use, with the objective of maximising farm profit. This scheduling method included a complex farm simulation model in the objective function, used decision variables to describe general management decisions, and used a custom heuristic method for optimisation. Existing optimal schedulers generally use stochastic dynamic programming which relies on time independence of all parameters except state variables, thereby requiring over-simplistic crop models. An alternative scheduling method was therefore proposed which allows for the inclusion of complex farm system models. Climate stochastic properties are modelled within the objective function through the simulation of several years of historical data. The decoupling of the optimiser from the objective function allows easy interchanging of farm model components. The custom heuristic method, definition of decision variables, and use of the Markov chain equation (relating an irrigation management strategy to mean water use) considerably increases optimisation efficiency. The custom heuristic method used simulated annealing with continuous variables. Two extensions to this method were the efficient incorporation of equality constraints and utilisation of population information. A case study comparison between the simulated annealing scheduler and scheduling using stochastic dynamic programming, using a simplistic crop model, showed that the two methods resulted in similar performance. This demonstrates the ability of the simulated annealing scheduler to produce close to optimal schedules. A second case study demonstrates the ability of the simulated annealing scheduler to incorporate complex farm system models by including the FarmWi\$e model by CSIRO in the objective function. This case study indicates that under conditions of limited seasonal water, the simulated annealing scheduler increases pasture yield returns by an average of 10%, compared with scheduling irrigation using best management practice. Alternatively expressed, this corresponds to a 20-25% reduction in seasonal water use (given no change in yield return).

Keywords: Irrigation scheduling, optimisation, simulated annealing, Markov chain

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1 INTRODUCTION

1.1 Context and Overview

This chapter gives a background to irrigation in NZ, identifies the need to increase efficiency, and discusses the role of irrigation scheduling in improving efficiency. The thesis objective, overview, and original contributions are presented.

1.2 Background

1.2.1 *Social and economic impacts of irrigation*

Irrigation within New Zealand (NZ) has important social and economic benefits. There are approximately 500,000 ha of irrigated land in NZ, the majority of which are on the Canterbury plains. Seventy-seven percent of all consumptive water allocation in NZ is for irrigation, with 64% of these irrigation takes in Canterbury and 23% in Otago. Approximately 60% of takes are from surface water, most of which are for community border-dyke schemes – some dating back 125 years. The remaining 40% of takes are from groundwater, for predominately spray irrigation systems (LE 2000a). The direct net contribution to NZ's GDP is approximately one billion dollars (Doak et al. 2004).

Ongoing irrigation development continues to increase the demand for water abstraction; the area of irrigated land doubling in the last two decades. At the same time, there has been a shift in values within communities towards increased protection of the natural environment and maintenance of biodiversity. Greater emphasis is now placed on leaving water within rivers, maintaining groundwater pressure to ensure adequate flow in low-land streams, and ensuring irrigation-induced land-use change does not compromise groundwater quality (Morgan et al. 2002). In many areas, the demand for irrigation water is greater than what can be supplied within these environmental constraints. This will place increased pressure on farmers to increase water use efficiency. In some areas, before any further irrigation development is allowed, the rural community as a whole may need to change its management practice to reduce nitrogen leaching to levels that can be assimilated by the aquifer. For example in some parts of Canterbury (Hanson 2002) and in the central North Island (EW 2003), nitrogen loading has become a controlling environmental constraint to future agricultural development.

1.2.2 *Improving on-farm irrigation efficiency*

Improving irrigation efficiency involves increasing the proportion of rainfall and applied irrigation water utilised by the crop. As a result, deep drainage is also reduced.

Efficiency can be improved in two ways. One approach is to change to a more efficient irrigation system. The potential efficiency of a system is primarily governed by (a) flexibility, (b) application uniformity, and (c) system capacity. These are further explained as follows:

- a) Flexibility is affected by both the irrigation system and the type of water supply. Flexible systems – such as centre-pivots with on-demand water supply – can more closely control the range of soil moisture levels, through applying small and frequent irrigation and varying the return period in response to rainfall and actual evapotranspiration. At the other end of the spectrum, highly inflexible systems – such as border-dyke systems connected to a community distribution scheme (where water may only be available a few days every month) – generally apply large irrigation depths and have little ability to change the return period in response to the climate.
- b) Application uniformity is primarily a function of the type of irrigation system. Given all other factors are the same, poor application uniformity can decrease efficiency by up to 15%, compared with more uniform systems (Section 2.2.5).
- c) System capacity influences potential efficiency, since systems with low system capacity have poor catch-up ability, thereby requiring more conservative (less efficient) irrigation scheduling.

The second approach to improve irrigation efficiency is to improve irrigation scheduling. Changing a system can require significant capital expenditure. In contrast, the cost of improved system management is relatively low. Improved scheduling can result in significant reductions in the total water used during a season.

1.2.3 Drivers for change

Historically there has been limited incentive for farmers in NZ to improve irrigation efficiency, since the on-farm variable/volumetric cost of water is relatively low, and there is limited regulatory control of non-point source contaminant leaching.

There are several factors that have resulted in a low variable cost of water:

- (a) The historic abundance of water available for abstraction and the regulatory allocation process. This historic abundance of water has, in the past, limited competition for water between irrigators, and has resulted in a regulatory allocation system where the maximum water available to a farmer over a season is far in excess of plant requirements. This results in little incentive for irrigators to use water efficiently, and locks up unused allocated water, making it unavailable for other consumptive use within a community (Bright 2006). Allocation of water on a first-come-first-serve basis, with long-term non-transferable consents, has also contributed to water having a relatively low market value.
- (b) The low cost to individual farmers of historic community storage facilities due to large government subsidies.

- (c) The high fixed costs of changing inefficient border-dyke and wild flooding systems. Many such systems have low operating costs (and therefore low variable water use costs), but use water inefficiently. The high capital cost of improving efficiency is a deterrent to undertake such works.

Increasing water scarcity has already increased the variable cost of water for recent irrigation developments. The cost of water source development has risen sharply in recent years, since the more economical options have already been utilised and because it is no longer the norm for large government subsidies to construct community schemes. The variable cost of water is generally highest for systems that required construction of storage facilities due to debt financing cost, or for supplies with high pumping costs.

Increased water scarcity and increased emphasis on environmental issues has already motivated regulatory authorities to reconsider water allocation processes. Some examples include: (a) Water allocation has recently been given a position of national prominence with the setting up of the Water Programme of Action, led by the Ministry for the Environment (MfE 2003); (b) Increasingly regional councils are imposing seasonal limits on irrigation water use; (c) A national standard is being proposed which would require compulsory measuring of actual water use (MfE 2006) (this seldom currently occurs); and (d) Research is currently being conducted on alternative institutional allocation arrangements for groundwater (Memon and Skelton 2006), and on water trading (Raffensperger and Milke 2005)

In addition to increasing water scarcity, a rapid increase in dairying since the mid 1990's has increased concern about the effects of nitrogen leaching on water quality. Increased nitrogen levels have already resulted in over-nitrification of certain surface water resources. There is also concern that nitrogen in groundwater may pose a public health risk and/or tarnish the pristine perception of very high quality groundwater resources such as the Canterbury Plains. Research is currently being undertaken to provide tools that can quantify sustainable nitrogen loading levels for individual aquifers (IRAP 2003). Increased irrigation efficiency reduces deep drainage, which reduces the mass of nitrogen leached. Also, in NZ virtually all irrigation water is low in dissolved salts; therefore, minimum leaching volumes are not required to counter soil salinity as is common in many parts of the world.

These drivers for change are expected to eventually increase the uptake of computer support irrigation scheduling tools, such as those developed in this thesis.

1.2.4 Irrigation scheduling methods

Irrigation scheduling requires making decisions as to when and where (which crops/paddocks) to irrigate. Good irrigation scheduling requires not only that irrigation be efficient, but that it also meets the economic objectives of being effective (LE 2000b).

The simplest method is to use conditional strategies (rules of thumb), which may incorporate simple parameters such as the maximum plant available water, the crop type, and the time of year. Often the rules will have no mathematical bases, relying only on an operator's experience. These conditional strategies are typical of most current irrigation scheduling in NZ. However, this approach can be both inefficient and ineffective, resulting in an excess of both water use and drainage, and a reduction in farm profit. One particular weakness to this approach is that irrigation scheduling is seldom adjusted in response to small or moderate rainfall events.

The next level of scheduling control is to maintain soil moisture within target soil moisture levels by directly monitoring soil moisture, and/or by predicting soil moisture through computer modelling. Computer programmes used in practice generally utilise a crop water use model like FAO 56 (Allen et al. 1998). Such programmes have been developed by California State University (Zoldoske 1990), Purdue University and Michigan State University (Joern et al. 1997) and the United Nations (FAO 1992), among others.

An advanced level of scheduling control is optimal irrigation schedulers. The previous two scheduling methods described above are unable to provide advice on how to schedule when water is scarce and/or when a restrictive farm system capacity forces prioritisation between crops. Water now becomes a limited resource that requires allocation in both time and space. In these situations optimum irrigation schedulers are of value. Optimal scheduling decisions are further complicated when rainfall during the season contributes significantly to soil moisture; as is the case in NZ where highly variable rainfall within the season can account for over 50% of the moisture supplied to the root zone. Most existing optimal irrigation scheduling algorithms assume a FAO 56 type crop water use model coupled with a FAO 33 type yield model (Doorenbos et al. 1979) – for example, the optimal scheduler of Córdova Rodriguez and Bras (1979). Optimal irrigation scheduling techniques are currently not used in practice. In a strategic paper commemorating 150 years of the ASCE Irrigation and Drainage Association, English et al. (2003) state that “*Irrigation optimization has been a subject of research for at least four decades, but to our knowledge no rigorous and systematic optimization procedures are being used in production agriculture today*”. These authors note that one reason for the lack of optimisation being used in practice is “*some of the [most well known] models of today, when applied to the problem of irrigation optimization, are found to have significant shortcomings*”. Jensen (1980) similarly commented that optimal control algorithms are seldom used in practice because of the limitations of the crop models incorporated in them, and because the variable cost of water has traditionally been low. The focus of this thesis is on overcoming these ‘*significant shortcomings*’.

1.3 Thesis Objectives

The objectives of this thesis were to (a) develop an on-farm irrigation scheduling algorithm, which provides advice on how to schedule irrigation between competing crops (when daily and seasonal water is limited), with an objective function of maximising profits; (b) test and compare this algorithm

against existing optimal scheduling algorithms; (c) incorporate the algorithm into a complex farm-system model; (d) assess the potential for this algorithm to increase profitability given water use constraints, using a realistic farm model; and (e) assess the potential for this algorithm to decrease seasonal water use given no change in average profitability, using a realistic farm model. The scheduling algorithm was specifically developed for Canterbury, NZ; however, the concepts could potentially be applied to most locations in NZ and internationally. The focus is on irrigation systems which have an on-demand water supply and a seasonal water allocation.

1.4 Thesis Overview

This chapter gave a background to irrigation in NZ, identifies the need to increase efficiency, and discusses the role irrigation scheduling improvements has to play in improving efficiency. The thesis objective, overview, and original contributions are presented.

Chapter 2 discusses farm system modelling requirements. Key questions answered are: *What are appropriate models within the context of irrigation scheduling for (a) irrigation systems, (b) irrigation water use, and (c) farm profit?* Nitrogen leaching modelling requirements is also discussed, although it is concluded that this is not a necessary modelling requirement for irrigation scheduling. Principal consideration is given to irrigated pastoral farms, since these are the major water users and sources of nitrogen leaching in NZ.

Chapter 3 discusses climate modelling requirements, with the objective of quantifying the stochastic properties of evapotranspiration (ET) and rainfall, and how variability in these two variables influences irrigation water use. It is discussed whether it is beneficial to use a cluster model in rainfall modelling, and whether trends or cycles with a duration of greater than one year are important. Historic climate timeseries from Christchurch Airport are used for most of the analysis. The daily reference ET timeseries, after seasonal standardisation, is modelled using mean values as a function of the time of year, a first order auto-regression model, and an auto-regression moving average model. Rainfall is modelled using a compound-Poisson model and a two-state Markov chain model. Similar models were used in previous optimal irrigation scheduling algorithms. The impact of specific stochastic climate properties on irrigation water use is quantified by using various combinations of ET and rainfall timeseries models in farm simulations. The potential for weather forecasting to reduce water use is also quantified.

Chapter 4 reviews the strengths and weaknesses of existing irrigation schedulers, while Chapter 5 gives an overview of common optimisation and heuristic methods in the context of their suitability for irrigation scheduling optimisation. Both chapters are in light of the constraints and modelling requirements of irrigation systems described in Chapters 1 to 3.

Chapter 6 presents a novel optimal irrigation scheduling method (termed the Simulated Annealing irrigation scheduler, or SA scheduler) that meets the modelling and constraint requirements set forth in Chapters 1 to 3. In addition to lessons learned from Chapters 4 and 5, experience is also drawn from two optimisation problems that are closely related to irrigation scheduling: rotational grazing and optimum farm management. Two specific components of the novel SA scheduler (a water use equation and a custom heuristic method) are developed separately in Chapters 7 and 8 respectively. The scheduler's performance is presented in Chapters 9 and 10.

Chapter 7 derives an equation relating irrigation strategies to mean irrigation water use. This equation is required within the novel scheduler; both when mapping between decision variables and irrigation strategies, and within the objective function simulation (to adjust irrigation strategies in response to variations between mean and simulated water use). Chapter 7 presents three possible methods for this equation: simulation of multiple seasons, an analytical relationship using Derived probability distributions, and Markov chains. The motivation for the latter two methods is computational efficiency, an important issue due to the large number of times this equation is used within the SA scheduler.

Chapter 8 develops a custom heuristic method for use within the SA scheduler. The design of this heuristic method and structuring of the optimisation problem in Chapter 6 are closely coupled for increased efficiency. The heuristic method optimises in continuous space, utilises both gradient and population information, and incorporates equality constraints in an efficient manner.

Chapters 9 and 10 demonstrate the performance of the novel scheduler through two sets of case studies. The first set uses simple soil and crop models, and compares the novel scheduler's performance with known optimum solutions provided by stochastic dynamic programming (Chapter 9), while the second case study incorporates more realistic farm system models (Chapter 10).

Chapter 11 concludes that the SA scheduler was able to overcome the principle limitation of existing optimal scheduling algorithms. Areas of further research previously identified in the thesis are summarized.

Following the thesis chapters, the glossary defines symbols, mathematical functions, abbreviations, and proper names. The Appendix contains an electronic copy of input data and source code.

1.5 Original Contributions

The principal original research contribution of this thesis is the development of an optimal irrigating scheduling method for multiple crops, capable of incorporating complex farm system models, and allowing for constraints on irrigation system capacity and seasonal water use limits. Secondary contributions include:

- A method for deriving compound-Poisson parameters from daily rainfall records (Chapter 3).
- Quantification of the importance of different stochastic properties of climate in the context of irrigation water use (Chapter 3).
- A method for optimally using weather forecasting information for irrigation scheduling (Chapter 3).
- A Markov chain equation for estimating future mean water use that has a computational efficiency two orders of magnitude faster than current simulation methods (Chapter 7).
- A modification of Press et al.'s (2002) simulated annealing algorithm for continuous variables, to allow inclusion of equality constraints (such constraints are common in a range of allocation optimisation problems, and not just within the specific optimisation problem considered in this thesis); and a modification to Press et al.'s method which allows population information to be utilised in optimisation (Chapter 8).
- A method for multi-crop optimal irrigation scheduling (which allows for both system capacity and seasonal water use constraints) which uses standard stochastic dynamic programming.

2 SYSTEM DESCRIPTION

2.1 Context and Overview

In order for an irrigation scheduling decision support tool to evaluate different irrigation management strategies, a model of the farm system is required. Key questions answered in this chapter are: *What are appropriate models within the context of irrigation scheduling for (a) irrigation systems, (b) irrigation water use, and (c) farm profit?* Answers are used in future chapters. Principal consideration is given to irrigated pastoral farms, since these are the major water user in NZ. Climate modelling requirements are discussed separately in Chapter 3. Section 2.2 describes the characteristics of irrigation systems, Section 2.3 the modelling requirements for irrigation water use, and Section 2.4 the modelling requirements for farm profit.

2.2 Water Supply and Irrigation Systems

2.2.1 Water supply

In NZ, approximately 60% (by area) of water supplies are sourced from surface water, mostly via long-standing community schemes (LE 2000a). The majority of community schemes are from run-of-river supplies and do not have any way of storing water over the season. Examples of community schemes that have storage impoundments are the scheme supplied from the Mannerburn reservoir in Ida Valley (Otago), servicing 5000 ha (Lloyd and Brown 2003), and the Opihi Augmentation Scheme supplied from the Opuha dam in South Canterbury, which services 15,000 ha (Agriculture NZ Ltd 2001). Community schemes generally use open canals for conveyance. Most of these schemes were constructed prior to spray irrigation becoming popular, with about 60% of development (by area serviced) occurring prior to 1950 (Agriculture NZ Ltd 2001). The legacy of wild-flooding and border-dyke irrigation has resulted in most of these schemes supplying water to the farm boundary on a roster system rather than a continual basis. For smaller farms, this means water may be available for only a few days per month. In addition to the roster delivery system, supplies are further interrupted by intermittent restrictions associated with minimum river flows.

Most of the remaining 40% of water supplies are sourced from groundwater. Most groundwater supplies have been developed more recently and supply predominantly spray irrigation systems. Generally, these supplies have separate bores for individual farms and are not connected to any community scheme.

Water takes of any significant volume in NZ require a water consent. Water consents may involve limits on the daily water use, and occasionally on seasonal water use. However, the latter are becoming more common as regulatory authorities aim to reduce pressure on groundwater resources and to encourage greater efficiency.

2.2.2 *Types of irrigation systems*

Approximately 60% of irrigation systems (by area) use either border-dykes or wild-flooding, while the remaining 40% are spray systems. Common spray systems for pastoral and cropping farming include rotary booms, hard and soft hose guns, centre-pivots, linear booms, and K-line. Common systems for horticulture are fixed sprinklers, micro and drip irrigation. These systems were classified by their control characteristics into three types: surface system, moveable spray system, and fixed spray and drip system. These are defined in Table 2-1.

Table 2-1: Irrigation systems classified by control characteristics.

Type of system	Control characteristics	Level of control	Examples
Surface	<ul style="list-style-type: none">• Discontinuous water supply• Restricted application depth• High application depth and long return periods	Low	Border-dykes, wild flooding
Moveable spray	<ul style="list-style-type: none">• On-demand water supply• Restricted application depth• Moderate application depth and return periods	Moderate	Rotary booms, linear booms, guns, K-Lines
Fixed spray (and drip)	<ul style="list-style-type: none">• On-demand water supply• Variable application depth• Small application depths and short return periods	High	Centre-pivots, fixed sprinklers, micro irrigation, drip irrigation

Surface systems are usually characterised by a discontinuous water supply and restrictions on the application depth. Generally, water is delivered to farms on a roster system and is available only a portion of the time. Large sections of a farm are irrigated simultaneously when water is available. Return periods¹ are typically 2-4 weeks. Overall, these systems have a low level of scheduling flexibility.

Moveable spray systems are usually characterised by an on-demand water supply and restrictions on the application depth. Systems are typically moved daily. Application depths can be partially reduced by increasing the speed of the unit, running the unit for only a portion of the day (however, this generally reduces the system capacity), or moving the units more than once a day. Return periods are typically 10-14 days. Overall, these systems have a moderate level of scheduling flexibility.

Fixed spray and drip systems are usually characterised by an on-demand water supply and a variable application depth. System capacity does not need to be lowered in order to reduce the application depths. The greatest level of control is achieved by applying small application depths with short return intervals. Overall, these systems have a high level of scheduling flexibility.

All irrigation systems have a system capacity limit – the maximum rate (averaged over the return period) that water can be supplied on-farm divided by the irrigated area, expressed in l/ha/day or

¹ Time period between consecutive watering of the same paddock.

mm/day. It is common for the peak daily plant water requirements to be greater than the available water; therefore, during peak times in the season, an irrigation manager will not be able to fully irrigate all paddocks, but must prioritise between different paddocks and crops.

2.2.3 Thesis scope restriction

Subsequent discussion and development of an optimal irrigation scheduling algorithm will be in the context of moveable spray and fixed spray (and drip) systems, since they have greater scheduling flexibility and will therefore gain the most benefit from such a tool. One distinctive control characteristic of these systems is access to an on-demand water supply. Therefore while systems with intermittent water supplies are outside the scope of this thesis, many of the ideas discussed and developed can be extended to systems with intermittent supplies.

Subsequently, it is assumed that all irrigators have a fixed application depth. This may appear restrictive for irrigators such as centre-pivots (which are capable of applying a range of irrigation depths), but, in practice, these irrigators will be designed for an optimal application depth. Decreasing the application depth is a trade-off between increased soil moisture control and reduced drainage, negatively offset by increased evaporation losses and stock management difficulties (since irrigation is not always possible around stock).

2.2.4 Terminology

Table 2-2 and Figure 2-1 present the irrigation system terminology used within this thesis.

Table 2-2: Irrigation system terminology as defined by the author.

Term	Definition	Examples
Irrigation unit		Rotary boom, big gun, collection of K-Lines (which are moved together), centre-pivot
Irrigation system	One or more identical irrigation units dedicated to a specific region of the farm.	Single centre-pivot, collection of rotary booms
Paddock	Area that can be irrigated in one day by a single irrigation unit	6 ha irrigation run for a rotary boom, 60 ha semi-circle irrigated by a centre-pivot
Block	Collection of Paddocks that have the same crop and soil properties, serviced by one or more identical irrigation units.	10 paddocks (60 ha) of pasture serviced by one rotary booms, single centre-pivot, 60 ha semi-circle irrigated by a centre-pivot
Farm portion	Collection of Blocks that share the same irrigation system.	3x60 ha Blocks of pasture, lucerne, and wheat (totalling 180 ha), serviced by two rotary booms

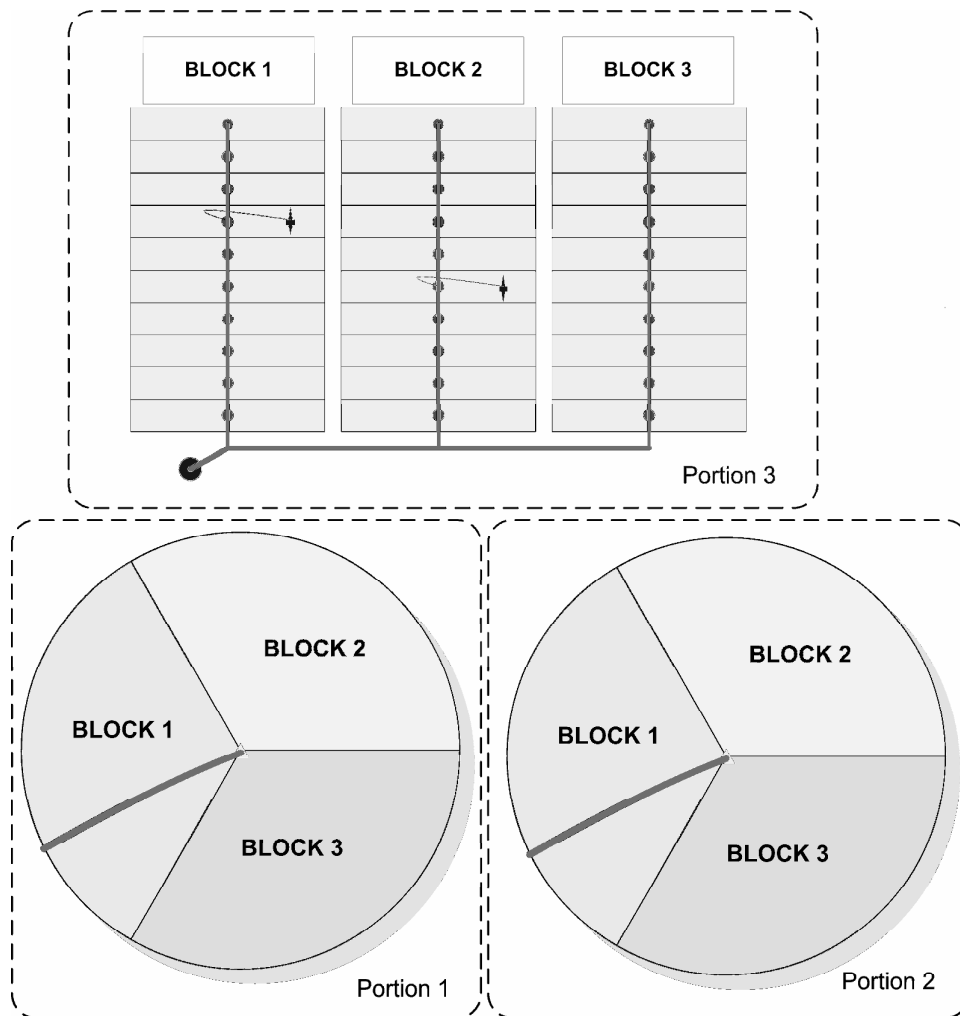


Figure 2-1: Illustration of terms *Paddock*, *Block* and (farm) *Portion*, as described in Table 2-2.

The illustrative farm in Figure 2-1 consists of three farm Portions. Two Portions are irrigated by centre-pivots and one Portion is irrigated by two rotary booms. The short return period of the centre-pivots results in each block being able to be irrigated in a single day. Therefore for these pivots, a Paddock and Block are synonymous. In contrast, the longer return period of rotary boom irrigators results in several Paddocks per Block. Each of the centre-pivots, and the collection of Blocks which all use the rotary boom irrigation system, constitutes a farm Portion.

The concept of Portions encapsulating complete irrigation systems is that good irrigation design will generally allow all systems to operate simultaneously at maximum capacity, and that each system will be assigned to service only particular areas of the farm.

The only dependency between the irrigation systems for each farm Portion is the seasonal farm water use constraint. This allows a decompositional approach to be used in irrigation scheduling. The decompositional approach involves finding an optimal scheduling regime for each farm Portion, assuming a particular seasonal water use limit. For each farm Portion, this process is repeated several times with different seasonal limits. From this, a relationship for each farm Portion can be obtained

between the seasonal water use and profit. These relationships can be used with a non-linear optimiser to optimally allocate seasonal water between farm Portions to maximise total farm profit. Sunantara and Ramirez (1997) used this type of decomposition to break a multi-paddock optimal irrigation scheduling problem into a series of single paddock optimal scheduling problems.

2.2.5 *Effect of spatial non-uniformity on application efficiency*

The principal application efficiency loss is generally caused from spatial application non-uniformity (Jensen 1980). The degree of spatial application uniformity can significantly affect application efficiency, particularly if the mean application depth is close to the soil moisture deficit². Bright (1986) derived a spray irrigation application efficiency model, assuming soil Total Available Water and soil moisture properties are uniform prior to irrigation, the spatial distribution of infiltration water has a Gaussian distribution, and ignoring evaporation losses. A non-dimensional solution of Bright's equation is presented in Equation 2-1. A range of typical values of Christiansen's coefficient of uniformity (UCC) (Christiansen 1941) for some NZ systems is presented in Table 2-3. Figure 2-2 plots Equation 2-1 over the typical range of UCC.

$$q = \frac{1}{2} \left(\text{Erf} \left(\frac{1}{\sqrt{\pi} (1 - \text{UCC})} \right) + \text{Erf} \left(\frac{1 - d^*}{\sqrt{\pi} (1 - \text{UCC}) d^*} \right) + \frac{1}{d^*} \text{Erfc} \left(\frac{1 - d^*}{\sqrt{\pi} (1 - \text{UCC}) d^*} \right) - (1 - \text{UCC}) \text{Exp} \left(- \frac{(1 - d^*)^2}{\pi (1 - \text{UCC})^2 (d^*)^2} \right) \right)$$

Where:

$$q = \text{Application efficiency} = \frac{\text{Applied water retained in the root zone}}{\text{Application depth}} \text{ (unitless)}$$

$$d^* = \frac{\text{Application depth}}{\text{Soil moisture deficit}} \text{ (unitless)}$$

UCC = Christiansen's coefficient of uniformity (Christiansen 1941) (unitless)

$$\text{Erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z t^{z-1} e^{-t} dt = \text{Error function}$$

$\text{Erfc}(z) = 1 - \text{Erf}(z) = \text{Complementary error function}$

Equation 2-1: Spray irrigation application efficiency model (Bright 1986), as a function of dimensionless application depth and spatial application uniformity.

² Soil moisture deficit = Soil moisture at field capacity minus available soil moisture

Table 2-3: Christiansen's coefficient of uniformity (UCC) for some NZ irrigation systems.

System	UCC (%)		Source
	Range	Typical	
K-Line	50-63	60	(Rout 2003)
Travelling gun	55-60	58	
Centre-pivot	85	85	
K-Line	50-57	55	(Edkins 2001)
Travelling gun	Not given	77	

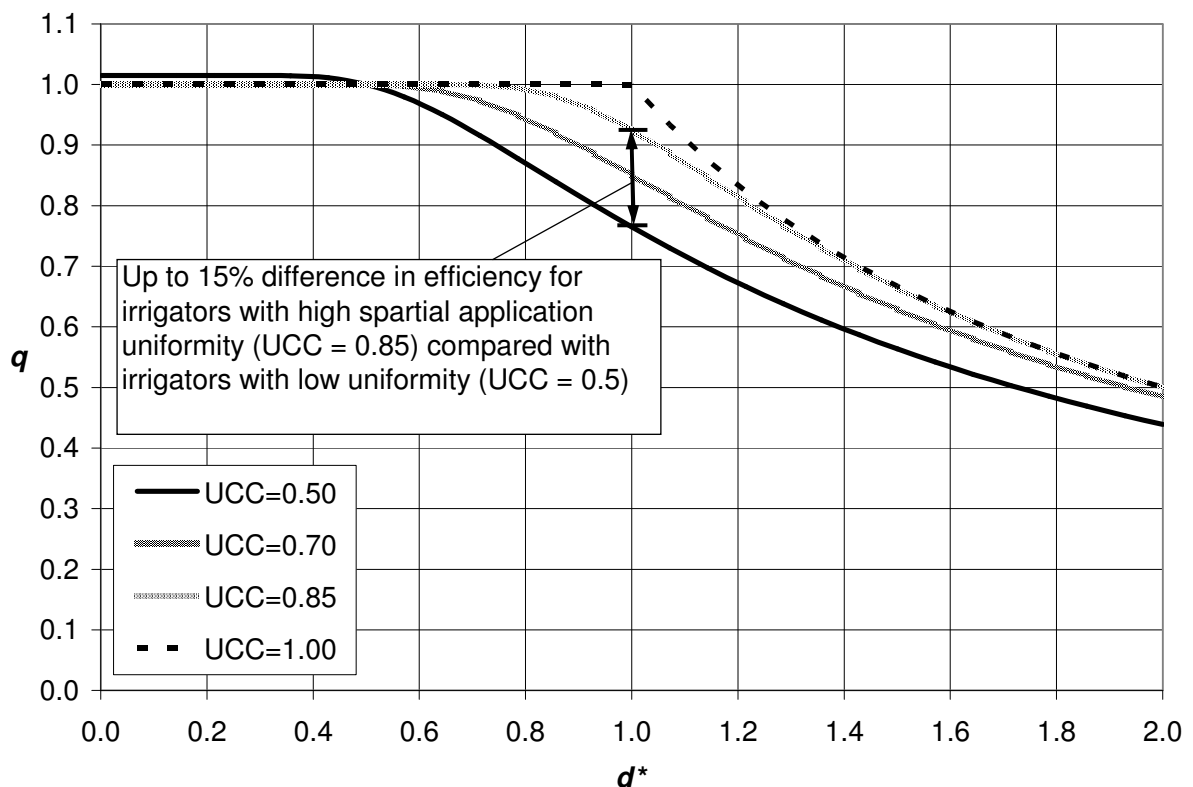


Figure 2-2: Spray irrigation application efficiency model (Bright 1986), as a function of dimensionless application depth (d^*) and spatial application uniformity (UCC).

In Figure 2-2, the application efficiency (q) of slightly greater than 1.0 for UCC=0.5 is a result of a mathematical simplification in the derivation of Equation 2-1, where it was assumed $(1-\text{UCC})^2$ is small relative to 1.0. This is not an ideal assumption for UCC values of less than about 0.7. This figure does highlight how significant spatial application uniformity is, with irrigators with poor uniformity characteristics up to 15% less efficient than more uniform systems, given the same application depth to soil moisture deficit ratio (d^*). The real difference between a uniform and non-uniform system can be even greater than this, since non-uniform systems require a higher irrigation depth to soil moisture deficit ratio in order to achieve the same adequacy of irrigation. Adequacy of irrigation is the (spatial)

proportion of the irrigated area where the soil moisture is above some minimum threshold. Non-uniformity results in a lower adequacy of irrigation since there is greater contrast between over-irrigated and under-irrigated regions (LE 2000b).

In practice soil moisture is never spatially uniform (as was assumed in Bright's application efficiency model derivation). As a result efficiency will be lower than is indicated in Equation 2-1 and Figure 2-2. Field quantification of soil moisture variability is more difficult than measuring irrigation application uniformity, and is seldom considered when modelling farm systems. Like irrigation application uniformity, soil moisture variability will have the greatest influence on application efficiency when the irrigation application depth is close to the mean soil moisture deficit.

The influence of non-uniform application is an important issue when modelling irrigation water use. All subsequent developments of an irrigation scheduling support tool in this thesis allow for Bright's application efficiency model to be included. However, in some simulation studies, an UCC value of 1.0 is used for simplicity.

2.2.6 *Other losses*

Water losses from sources other than spatial non-uniformity include distribution (network) and evaporation losses. Distribution losses are low for well maintained pipe systems (McLean et al. 2000), but are more significant for open-channel systems. Spray evaporation losses (prior to water reaching the canopy) are dependent on the spray droplet size distribution, weather conditions, and water temperature. McLean et al. (2000) measured about 7% spray evaporation losses from a Big Gun (which produces small droplets), but negligible losses under a centre pivot (which produces large droplets). Jensen (1980) stated spray evaporation losses are typically less than 2% and canopy evaporation losses (following irrigation) less than 5-8%, concluding evaporation losses are small compared with spatial non-uniformity losses.

2.3 Irrigation Water Use

A variety of soil moisture models exist in literature, ranging from simple water balances to detailed mechanistic models based on Richards' equation. For calculating irrigation water use the former are often adequate, although dual layer water balance models are preferential to single layer models (Woodward et al. 2001). Common water balance models have one or more buckets (reservoirs) representing the moisture within soil layers. The capacity of these buckets is equal to the field capacity of each layer. Overflow from one bucket to the next (or to deep drainage) occurs when field capacity is exceeded. Figure 2-3 illustrates a dual layer water balance model.

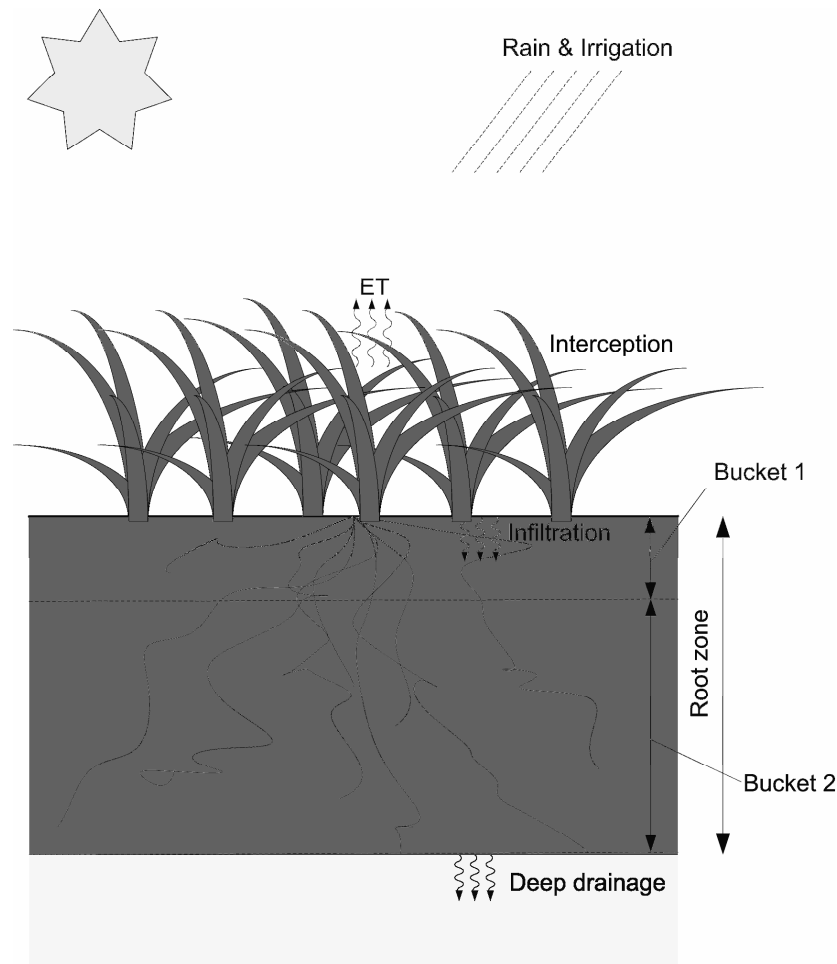


Figure 2-3: A dual water balance model, where the change in moisture stored in each bucket (reservoir) is equal to infiltration minus drainage and ET.

ET is commonly modelled using either the single or dual crop coefficient FAO 56 method (Allen et al. 1998). The dual crop coefficient method splits ET into the transpiration and evaporation components. Accounting for these two components separately can make a significant difference in water use estimates during crop development, when the ground is not fully shaded by the canopy, but has much less of an effect on fully developed crops such as perennial pasture.

Dual layer water balance models have been shown to give good agreement with field monitoring of soil moisture (Woodward et al. 2001). Single layer water balance models can overestimate moisture stress and therefore underestimates water use, particularly for deep soils or under frequent irrigation. A simulation study was undertaken by the author to quantify the differences in water use estimates between the single and dual layer models. Two different soils were simulated, each using the two alternative models, and for two alternative irrigation strategies. Parameters for this study are presented in Table 2-4 and the results are in Table 2-5. Soil 1 had a Total [plant] Available Water (TAW) of 60 mm while Soil 2 had a TAW of 120 mm.

Table 2-4: Parameters for the case study simulations illustrating the difference in simulated irrigation water use between a single and dual layer soil water balance model.

Parameter	Single layer soil model	Dual layer soil model
Plant model description	Single crop coefficient FAO 56 crop model after Allen et al. (1998). Moisture stress proportional to the greater of (1) the fraction of available water (FAW) of the top soil layer, or (2) the average FAW of all layers.	
Soil moisture model description	Single layer	Dual layer, after Woodward et al. (2001)
Crop	Pasture – constant rooting depth and crop coefficient of 1.0	
Soil 1 TAW	60 mm	Layer 1 – 30 mm Layer 2 – 30 mm
Soil 2 TAW	120 mm	Layer 1 – 45 mm Layer 2 – 75 mm
Soil [plant] Readily Available Water	50% of TAW	
Irrigation depth	Soil 1 – 30 mm Soil 2 – 60 mm	
Irrigation season	1 September – 31 March	
System capacity	8 mm/day (unconstraining)	
UCC	1.0	
Trigger soil moisture level	Variable, as noted in Table 2-5	
Case study irrigation years	Historical Christchurch climate from June 1960 to May 2004, as per Table 3-1	

Table 2-5: Simulated irrigation water use from single and dual layer soil water balance models, as a function of TAW and the irrigation trigger soil moisture level.

TAW (mm)	Trigger soil moisture level as % TAW	Mean seasonal water use (mm)	
		Single layer soil	Dual layer soil
60	0.3	507.9	558.1 ⁽¹⁾
60	0.5	616.1	616.1
120	0.3	460.5	526.1 ⁽²⁾
120	0.5	563.7	563.7
(1) 10% greater water use than the single layer soil			
(2) 14% greater water use than the single layer soil			

Table 2-5 shows that (as expected) when water stress does not occur (Trigger soil moisture level >0.5), simulated water use are identical between the single and dual layer water balance models. However, for deficit irrigation, the simulated water use from both these models varied by 10-14%, with higher predicted water use using the dual-layer model.

2.4 Farm Profit

2.4.1 General crop response to water stress

Crop physiological responses to soil moisture deficits (and its effects on yield quantity and quality) are more complex than estimating irrigation water use, requiring more detailed crop and soil models. Plants modify their biochemistry, physiology, growth, and development in response to a reduction in

soil water availability. Even at relatively low stress, plants may restrict stomata aperture and growth, indicating that biochemical responses (and not only hydraulic effects) influence a plant's response to water availability (Press et al. 1999). Concise crop-yield models, such as the coupling of FAO 56/FAO 33 (Doorenbos et al. 1979, Allen et al. 1998) over-simplify how plant development during the season is affected by soil moisture. The timing of harvest, quality and quantity of yield are all factors that affect farm profitability; therefore in order to account for these various factors, relatively complex models are required.

2.4.2 *Pastoral systems*

2.4.2.1 Predominance

In NZ, pasture is the principal agricultural crop, occupying 75% of the total agricultural area. The value of produce derived from pastoral farming – dairying, meat, wool and leather – accounts for 70% of all agricultural exports, with most of the remaining 30% derived from forestry (20%) and horticulture (9%). Exports from pastoral farming are about \$12.5 billion per year, or 40% of NZ's total exports (Statistics NZ 2003).

Eighty percent of irrigation occurs on pastoral land, half of which is for dairy farming and half for sheep, beef and deer farming. However, only 7% of the area used for dairying is irrigated, and 1.3% of the area of sheep, beef and deer farms is irrigated. The remaining irrigation water is used for horticulture (15%) and arable cropping (5%) (Statistics NZ 2003). Most of the irrigation occurs in Canterbury (64%) and Otago (23%) (LE 2000a).

The emphasis within this thesis is on pastoral farming because pastoral farming is the current predominant water user; however, this emphasis does not imply that irrigation of pasture is in any way more important than irrigation of other crops. In fact, there is a strong case for the reverse being true. Pasture has the highest water demands of virtually any crop. Under dairy farming, it has one of the higher nitrogen loadings to groundwater. Also, it is not the highest value crop, particularly when value is expressed in terms of yield value divided by seasonal water use.

2.4.2.2 Response to water stress

NZ pasture is a composite of forage grasses and legumes. The most common species in intensive farming are perennial ryegrass and white clover. For less intense farming, other grass species (browntop in particular) become more dominant due to their greater ability to tolerate low fertility and drought conditions (White and Hodgson 1999).

Pasture is generally more able to accommodate variations in environmental conditions than horticultural or arable crops. This is due to the continual cycling of tillers, with new tillers replacing senescencing tillers. Tillers damaged by environmental stress will eventually be replaced by new tillers (White and Hodgson 1999). In contrast, horticultural and arable crops generally have a determinant (rigid and predefined) seasonal life cycle. Environmental stress may affect critical stages

of crop development (depending on stress timing) significantly affecting the final yield quality and quantity. Furthermore, the economic value of crops for human consumption is much more sensitive to crop quality than pasture, which is for livestock consumption.

Perennial ryegrass is the most widely sown pasture species due to its high productivity, nutritional value, and persistence. However, it is shallow rooted and, in dry conditions, will be out-competed by deeper rooted species (such as browntop). These latter species tend to have lower yields and/or lower nutritional value (White and Hodgson 1999). Therefore, changes to irrigation management may result in changes in species composition.

The timing of pasture growth, and pasture quality impact on farm profitability, and are important factors when assessing optimal management (Snow 2004). Furthermore, the relationship between pasture quality and livestock performance is not linear. For example, a 15% difference in the energy per unit dry matter of pasture may make a 25% difference in animal output (Bywater 2006). As such, while crop quality may not be as critical in pasture as it is for certain arable crops, it is still an important factor when assessing the economic advantages of changing irrigation management strategies.

Short periods of moisture stress do not affect pasture growth rates upon restoration of the soil moisture. However, longer dry periods may result in either an increase or a reduction in normal unstressed growth rates when soil moisture is restored. If pasture can be kept alive, pasture growth can actually increase above normal levels if water is applied, following a dry period. This is due to a sudden release of nutrients, particularly nitrogen (Fraser 2004). If the soil is too dry, plant death will begin to occur, thus delaying growth when soil moisture is restored. This can be partially mitigated if water stress develops gradually, allowing drought resistance to develop through increased internal osmotic pressure.

An example of delayed growth following prolonged dry periods is the typical irrigation practices in Ida Valley (Otago, NZ), where irrigation events are six weeks apart. Soil moisture deficits develop after 10-12 days, followed by a month of water stress, resulting in partial plant death. When the next irrigation event does occur, there is a 4-5 day delay before grass starts to grow again, resulting in plant growth only occurring about seven days every six weeks (Fraser 2003).

An example of modelling of water stress for NZ's pastures is McCall's (1984) pasture growth senescencing and decay model. In this model, growth inhibition begins at soil moisture levels as high as 60-80% of the TAW; growth inhibition due to soil stress is dependent on the potential ET demand. Under high potential ET demand, there is some level of growth restriction even at high soil moisture contents. At low potential ET, unrestricted growth occurs until the available soil moisture reaches 60% of field capacity. Provided that the duration and severity of water stress is not too great, growth will return to maximum levels shortly after soil moisture replenishment. However, during long

periods of high water stress, the pasture begins to die, since growth is not occurring as fast as senescencing. At soil moistures below 20%, the rate of senescencing is increased, accelerating plant death. On restoration of soil moisture, initial growth will be below maximum levels, due to a reduction in the (green) leaf area index.

2.4.2.3 Livestock dynamics

For pastoral farming, it is important not only to consider the total pasture production in a season, but also the timing of feed availability and quality of feed. Both the quantity and quality of feed impact on animal performance. Periods of feed shortage can be mitigated through supplementary feeding; however, this is generally a more expensive food source than direct grazing. Alternatively, excess pasture growth beyond stock requirements may actually have a negative effect on farm production due to a reduction in pasture quality. Sometimes the timing of animal production, such as having lambs ready for the abattoir pre-Christmas, can have a significant impact on profitability.

2.4.2.4 Pastoral farm modelling

In NZ, the two main pasture models used are the McCall model and the Johnson/Woodward model. In addition to these, a complete farm system model termed FarmSim is currently being developed by Lincoln Ventures Ltd, with model component contributions from Lincoln Ventures Ltd, Crop and Food, AgResearch, Dexcel, Landcare Research and ESR (IRAP 2003). The scheduled release date for this software suite is 2008. Outside of NZ, sourcing models is difficult since NZ's pastoral farming system is distinctly different than most overseas sheep, beef and dairying systems. Hence, many of the farm system models from other nations, particularly from the USA and Europe, are not applicable in NZ.

An overseas farm system model which is suitable for NZ pastoral systems is FarmWi\$e from CSIRO Plant Industry, Canberra, Australia. There are a number of major advantages to this model, including (a) field verification on pastoral farms that are similar to typical NZ pastoral farms, (b) several peer-reviewed publications, (c) a track history and widespread use by the farming community as a real-time decision support tool, and (d) modern software architecture. In contrast, the three NZ models mentioned above do not have a track history of use as an on-farm decision support tool, and the McCall and Johnson/Woodward models use legacy software engineering techniques, such that incorporating them into an optimisation procedure is difficult. A brief description of these four models is provided below. The model from CSIRO Plant Industry is used in the case study in Chapter 10.

McCall pasture model

The McCall pasture model was initially developed for sheep farming in the North Island hill country (McCall 1984) and was later used within the Dexcel dairy farm model (McCall and Bishop-Hurley 2003). It was the principal pasture model used by AgResearch in the mid-1980's (Snow 2004). This empirical model predicts the rate of new growth, senescence and decay at a paddock scale. The

influences of spring flush, air temperature, soil moisture and leaf area index are accounted for in a multiplicative manner. Soil fertility is incorporated into a single parameter. Nitrogen uptake is not modelled. There are some concerns with model predictions in dry climates (Clark 2004). The model has the advantage that it is relatively simple and has some field verification for sheep and dairy operations in NZ.

Johnson/Woodward pasture model

The semi-mechanistic Johnson pasture model predicts the growth, development, senescence and decay of tillers (Johnson and Thornley 1983, 1985). Tiller age is modelled by four pools. The model accounts for all the environmental factors included in the McCall model, but in a more mechanistic manner. Nitrogen uptake is modeled, and pasture quality can be better estimated through the ages of tillers. This model was used in researching the potential effects of climate change on pasture production in NZ (Warrick et al. 2001). In a separate study, Clark (2004) found this model satisfactorily predicted seasonal dryland growth.

Johnson's pasture model, as adapted by Woodward (1997), until recently was the principal pasture model currently used by AgResearch. It has been used both within the Lincoln University LincFarm sheep system model and the Dexcel dairy system model. Three years of field verification have been cited for the LincFarm model, which performed reasonably well under both dryland and irrigated conditions at Winchmore (Canterbury, NZ). The model has been used for researching the effects of spatial variability in soil fertility, urine and dung deposits (Clark 2004). It was also used by Crop and Food research concerning land use change and intensification (Zyskowski 2004).

FarmSim, by Lincoln Ventures Ltd

FarmSim is a complete farm system model currently being developed, with model component contributions from Lincoln Ventures Ltd, Crop and Food, AgResearch, Dexcel, Landcare Research, and ESR (IRAP 2003). The scheduled release date for this software is 2008. Details of pasture and crop components under development by AgResearch have yet to be confirmed. The soil component has been confirmed; however, details of this model are not publicly available (Good 2007). The original purpose of this software was to assess the impact of land-use intensification on nitrogen levels within an aquifer.

FarmWi\$e, by CSIRO Plant Industry

CSIRO Plant Industry has produced a series of analysis and decision support tools, as part of their GrazPlan programme, for temperate Australia pastoral farming systems (Donnelly et al. 1997, Freer et al. 1997, Moore et al. 1997, Donnelly et al. 2002). The three principal tools are GrazFeed, GrassGro, and FarmWi\$e. GrazFeed is a decision support tool for grazers and their advisors, and is used to manage pasture and supplementary feed for sheep, beef and dairy farming. It has gained widespread industry acceptance since its release in 1990. GrassGro, which incorporates a more complex farm system model, is a general decision support tool for pastoral farming, and is used to assess the impact

of different management strategies under variable climatic conditions. GrassGro was released in 1997. FarmWi\$e is the most recent and generalised tool within GrazPlan, and provides a generic modelling environment for incorporating a variety of farm components. In addition to the modelling components incorporated into GrassGro, CSIRO Plant Industry's common modelling framework allows for the addition and/or interchanging of third party components (CPI 2005, Moore et al. 2005, CPI 2007). Advantages of using FarmWi\$e for modelling farm profit within an irrigation scheduling decision support tool includes the following:

- The model accounts for the effects of water stress on pasture growth and is applicable under both dryland and irrigated conditions.
- The model has been field verified for pastoral farming systems that are similar to typical NZ pastoral systems.
- There are peer-reviewed publications of model components.
- There is a track history of model components and related decision support tools having widespread acceptance and use by the farming industry (in Australia).
- The model has modern computer engineering architecture

Because of these advantages, FarmWi\$e is used for the case study (which uses the SA scheduler) in Chapter 10.

2.5 Conclusions

An optimal irrigation decision support tool must allow for constraints on both daily water use (system capacity) and seasonal water use. Within the context of this thesis, the development of such a tool has been limited to systems that have an on-demand water supply. The main on-farm water losses are generally from deep drainage, which is a function of the spatial application uniformity.

Irrigation water use is commonly estimated from the FAO 56 method (Allen et al. 1998) either with a single or dual ET crop coefficient, and with either a single or dual layer soil water balance model. A dual crop coefficient, dual layer soil is preferable.

Farm profit is more difficult to estimate than irrigation water use, since crop physiological responses to soil moisture deficits and their effects on yield quantity and quality require more complex models. One suitable model is FarmWi\$e by CSIRO Plant Industry. FarmWi\$e is used in the SA scheduler case study in Chapter 10.

3 CLIMATE CHARACTERISTICS

3.1 Context and Overview

Climate is the principle factor governing irrigation water use. The highly stochastic nature of climate (particularly rainfall) is also the primary factor affecting water use and farm profit variability. The objective of this chapter was to quantify the stochastic properties of ET and rainfall, and show how these two influence irrigation water use. This was important when considering the strengths and weaknesses of previous optimal irrigation schedulers. It also allows decisions to be made about what climate models are appropriate for the SA scheduler and for the Water Use Equations derived in Chapter 7. Specifically, results showed whether it is beneficial to use a cluster model in rainfall modelling, and whether trends or cycles with a duration of greater than one year are important.

Qualitative aspects of NZ's climate are discussed, including its geographical setting, irrigated regions, and long-term cycles and trends. The remainder of the chapter focuses on quantifying the stochastic properties of ET and rainfall, and how these characteristics in turn affect irrigation water use. Christchurch Airport (Canterbury Plains, NZ) historic climate timeseries from 1960 to 2004 was used in most of the analysis. The daily reference ET timeseries, after seasonal standardisation, were modelled using mean values as a function of the time of year, a first order auto-regression model and an auto-regression moving average model. Rainfall was modelled using a compound-Poisson model and a two-state Markov chain model. The former model assumes no correlation between storm events, while the latter accounts for short-term correlation in rainfall occurrence, but does not account for correlation in rainfall depths. Similar models were used in previous optimal irrigation scheduling algorithms. The impact of specific stochastic climate properties on irrigation water use was quantified by using various combinations of ET and rainfall timeseries models in farm simulations. The potential for weather forecasting to increase irrigation efficiency was also quantified.

3.2 Background

3.2.1 *Geographical setting*

NZ's climate is influenced by its location in the southerly mid-latitudes where winds are predominantly westerly, by the moderation from the surrounding oceans, and by the mountains oriented north to south along the length of the country. The latter results in significant differences between the western windward region and the eastern leeward districts (particularly in the South Island), affecting wind patterns, precipitation, temperatures and cloud cover. The combination of mountainous terrain and differential heating of land and sea leads to complex local circulation

patterns, including land and sea breezes near the coast and strong katabatic winds³ close to the mountains. The overall result of these influences is limited seasonal temperature variations but much greater spatial variation in precipitation, in particular sharp east-west gradients in parts of the South Island. Climate contrast tends to be more marked from east to west than from north to south. The western slopes of the ranges are much cloudier and wetter than the eastern sides (Hobbs et al. 1998). For example, mean annual rainfall in parts of the western South Island exceeds 12,000 mm, but is only 300 mm 100 km away in Central Otago, which is to the lee of the Southern Alps.

Climate variability is mostly driven by successions of anticyclones and depressions in the westerlies, bringing alternating short periods of settled and unsettled weather. Typically, these cycles have a scale of one week. Sometimes blocking of these weather systems can lead to more persistent dry or wet spells (Sturman and Tapper 2006).

3.2.2 Irrigation regions

Irrigation in NZ is concentrated on the dry east coast of the South Island, with 85% of irrigation occurring in Canterbury and Otago. Rainfall in Canterbury and Otago ranges from 300-800 mm and annual reference ET ranges from about 900-1000 mm. Typical summer daytime temperatures range from 15-25° C, and winter temperatures range from 5-15° C (NIWA 2007). Rivers fed from the wet west coast of the South Island supply water to the dry east coast. Moderate temperatures means virtually all precipitation during the irrigation season is in the form of rainfall rather than snow.

3.2.3 Long-term cycles and trends

Climate can be considered a random stochastic process composed of random components with various different time scales (also known as ‘thick’ and ‘thin’ sub-systems). The timeframe of each component is governed by the amount of storage capacity (or ‘thickness’) in the sub-system that drives that particular variability. For instance, since the atmosphere has relatively limited storage capacity (in the form of spatial variations in moisture, heat, or kinetic energy), oscillations driven by atmospheric processes (such as cyclone and anti-cyclone systems) have a short timeframe – less than about two weeks. In contrast, ‘thick’ sub-systems such as oceans and ice sheets, which have a large amount of storage capacity (in the form of spatial variations in saline concentration and temperature), drive long-term climate oscillations. Some oscillations from such thick systems can have timeframes in the order of decades or longer (Dobrovolski 2000, Harrold 2002).

One such ocean oscillation is the El-Niño Southern Oscillation (ENSO), the main variable that has been correlated with large-scale variations in rainfall and temperature in Australia and NZ. The state of the ENSO is commonly expressed through the Southern Oscillation Index (SOI), which is a measure of the strength of the Walker circulation that moves air from the north-west to south-east of

³ A katabatic wind is wind generated by air that is cooled as it passes over a mount/slope, and then slides down the slope under the influence of gravity.

the Pacific. The Walker circulation breaks down during an El-Niño event (low SOI), and intensifies during a La-Niña event (high SOI). Negative SOI values are related to anomalous south-west airflow, and positive values result in anomalous north-easterly flow. Therefore, under El-Niño conditions, the whole country should be cooler, with the west wetter and the east dryer. In perspective, correlation with the SOI explains only about 10-25% of the variance in temperature and rainfall. The SOI does allow some prediction of seasonal weather patterns, due to the longer time scale of the ENSO. For example the first seasonal weather prediction was made by the NZ Meteorological Service in December 1982 during the major 1982/1983 La-Niña event (Sturman and Tapper 2006).

The possibility of long-term changes in climate (resulting not from natural ‘thick’ systems, but rather from human induced increases in CO₂) has been a topic of intense scientific and political debate over the last two decades. Currently, the commonly accepted view is that CO₂ emissions are affecting climate, and that this will result in a 1 - 6°C increase in average temperature by 2100, relative to 1990 (McCarthy et al. 2001). Predictions of local effects on NZ vary, with some predicting a decrease in the strength of the westerlies (Hobbs et al. 1998), while others predicting an increase in the strength of the westerlies (Wratt 2006). Generally, increased temperatures are expected to result in increased precipitation (Wratt 2006). Possible increasing ET in irrigated regions due to increased temperature could therefore be partially or fully offset by increased precipitation in these regions. For example, a study of the potential impact of climate change on Southland’s irrigation demands predicted that increased ET levels were more than offset by increased precipitation (Morgan and Evans 2003).

3.3 Climate Timeseries

Details of the historic timeseries used in the majority of analysis in the remainder of this chapter are presented in Table 3-1.

Table 3-1: Christchurch Airport (Canterbury Plains, NZ) historical rain and reference ET daily timeseries.

Parameter	Description	Source
Historic daily rainfall series	Christchurch Airport (Station H32451), 1/06/1960-31/05/2004	NIWA’s National Climate Database (NIWA 2007)
Historic daily reference ET series	Christchurch Airport (Station H32451), 1/06/1960-31/05/2004 (calculated from Penman-Montheith equation)	NIWA’s National Climate Database (NIWA 2007)

3.4 Evapotranspiration

Timeseries and statistical analysis was used to quantify the stochastic properties of Christchurch Airport historic reference ET. This analysis was used in Section 3.6, to quantify the influence of different stochastic components on irrigation water use.

3.4.1 Factors affecting evapotranspiration

ET occurs when liquid water from a vegetated surface is converted into water vapour. The rate is controlled by the availability of energy at the evaporating surface, stomatic resistance, and the ease with which the vapour can diffuse into the atmosphere. Weather, crop characteristics, and farm management are all factors that affect ET. Evaporation of water requires relatively large amounts of energy, either in the form of radiant energy or sensible heat⁴. The amount of energy available is the primary factor governing ET rates. For Canterbury, in summer, approximately 95% of the energy used for ET is obtained from radiation, while in winter, 50% of the energy is obtained from radiation and 50% from sensible heat⁵. Usually, reference ET is estimated from climate data. The Penman-Montheith equation is a commonly use method for estimating reference ET, which is physically based and has been shown to offer superior estimates in a range of climates compared with other common equations (Allen et al. 1998). The main terms in the Penman-Montheith equation, for daily estimates of reference ET, are presented in Equation 3-1.

$et_o \approx \frac{0.408 \Delta_v r_n + \frac{900}{temp} \gamma (v_s - v_a) w}{\Delta_v + \gamma (1 + 0.34 w)}$
<p>Where:</p> <p>et_o = Reference evapotranspiration (mm/day)</p> <p>$temp$ = Temperature [°K]</p> <p>r_n = Net radiation at the crop surface = f(extraterrestrial radiation, cloud cover, $temp$) (MJ m⁻² day⁻¹)</p> <p>w = Wind speed at 2m height (m/s)</p> <p>v_s = Saturation vapour pressure (kPa)</p> <p>v_a = Actual vapour pressure (kPa)</p> <p>Δ_v = Slope of saturated vapour pressure curve = f(temperature) (kPa/°C)</p> <p>γ = Psychometric constant (kPa/°C)</p>

Equation 3-1: Penman-Montheith equation with minor terms omitted, modified from Allen et al. (1998).

⁴ Sensible heat is heat from the ambient air temperature.

⁵ Estimated by the author from magnitude of relevant terms in the Penman-Montheith equation given Christchurch climate timeseries.

The two main factors affecting reference ET (ET_o) are radiation and temperature. For mid to high latitudes, radiation has a pronounced seasonal cycle due to the annual sinusoidal cycles in the amount of extraterrestrial radiation. In NZ, mean temperatures also follow closely to an annual sinusoidal pattern, driven by the annual solar radiation cycle (Figure 3-1). Thermal mass results in a lag of about one month between maximum radiation and maximum mean temperatures.

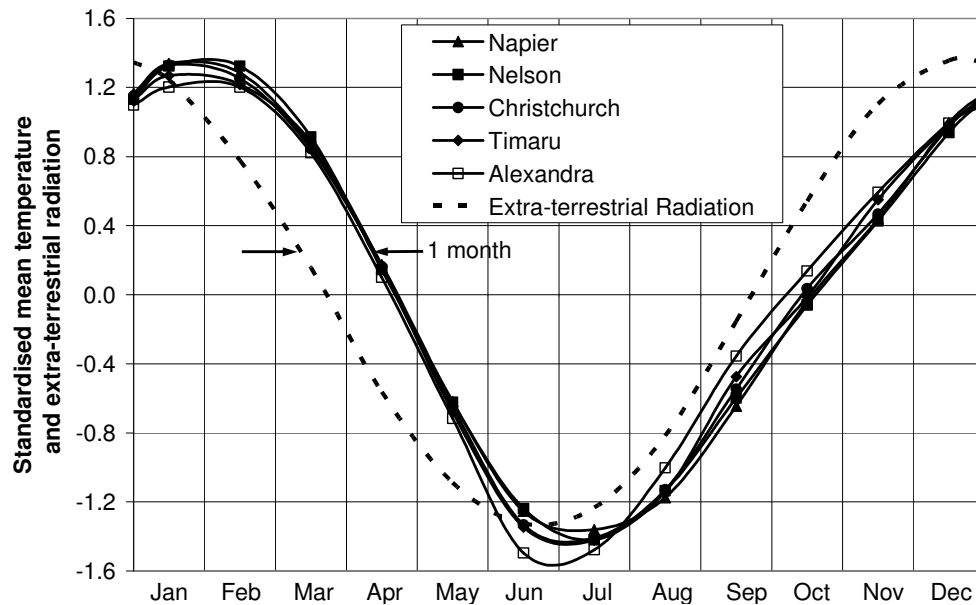


Figure 3-1: Annual sinusoidal temperatures cycles driven by extra-terrestrial radiation cycles, for several NZ towns which a representative of irrigated regions in NZ.

Mean monthly temperatures for Figure 3-1 were obtained from NIWA (2007) and are for the period from 1971 to 2000. Locations were chosen to be representative of irrigated regions in NZ. Extra-terrestrial radiation was estimated from the method of Allen et al. (1998)⁶.

3.4.2 Auto-regression modelling

Low order auto-regression (AR) and auto-regression moving average (ARMA) models are widely used for modelling stationary daily timeseries after seasonal standardisation (Maidment 1993). Previous optimal scheduling algorithms have modelled ET_o with mean values as a function of the time of year (Córdova and Bras 1979, Ramirez and Bras 1985, Sunantara and Ramirez 1997), and with a first order AR model (Rhenals and Bras 1981).

Rhenals and Bras (1981) used a log-normal transformation to remove skewness, and standardised to remove seasonal trends. Values for the lag-one serial Pearson's correlation coefficient ranged from 0.3-0.6, varying with the time of year. They found that modelling ET_o using an AR model in place of

⁶ Calculations assuming Christchurch's latitude is plotted. The variation in standardised radiation, when assuming latitudes of other locations in NZ, was less than 2%.

time varying means, resulted in minimal improvements for scheduling irrigation given their particular case study. One reason for their lack of benefit is likely due to the very high soil TAW (over 600 mm) used in their case study, which is much higher than typical Canterbury Plain soils.

The mean and standard deviation for Christchurch ETo are plotted as a function of time in Figure 3-2. The sinusoidal pattern can be attributed to the sinusoidal pattern in both extra-terrestrial radiation and mean temperature.

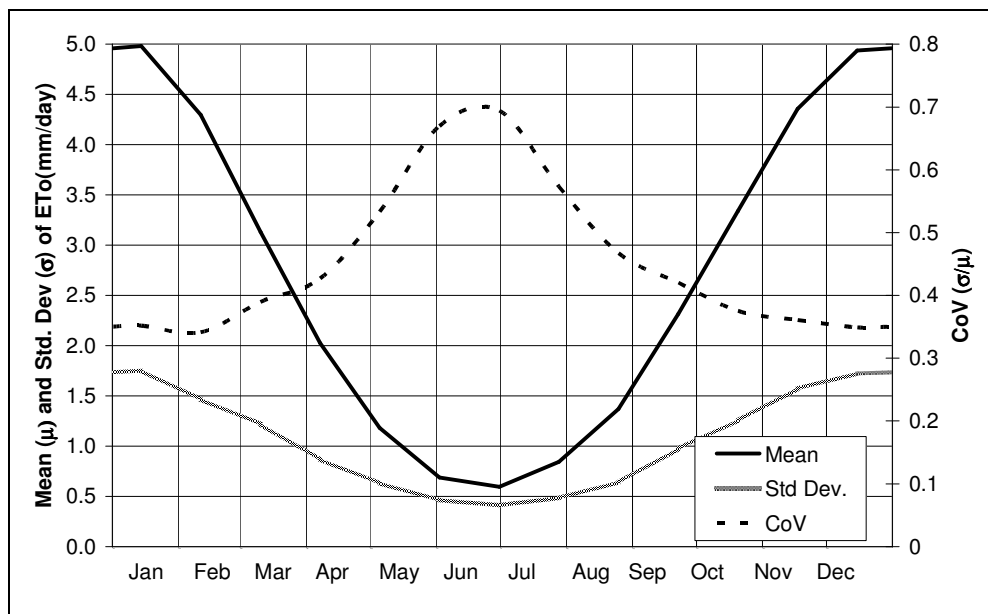


Figure 3-2: Seasonal mean and standard deviation variations in ETo at Christchurch Airport.

Christchurch daily ETo timeseries was standardised to remove seasonal trends in the mean and variance. Seasonal trends are principally associated with the annual sinusoidal cycles in extra-terrestrial radiation and temperature. Variance is principally associated with cloud cover variations and, to a lesser extent, fluctuations in temperature. The timeseries was standardised to the form $(ETo - \mu)/\sigma$. Firstly, the timeseries was divided into Time Aggregation Periods (TAPs), where each TAP corresponded to a particular period of the Julian year. Mean ETo (μ) was calculated for each TAP. Secondly, the standard deviation (σ_i) of $(et_o - \mu_{TAP_i})$ for each TAP_i was calculated (where et_o is daily reference ET).

The length of the TAPs affects the sample size for estimating the population mean and standard deviation for each TAP. Standard deviation estimates were particularly sensitive to random noise when the sample size was small. However, if the TAP was too long, seasonal trends were unnecessarily smoothed out. A TAP length of four weeks was found to be a good compromise, resulting in enough smoothing to remove most of the random noise from σ , without excessive

smoothing of seasonal trends⁷. This conclusion also applies if the available timeseries were much shorter than 44 years. Linear interpolation between the mid-points of the TAP was used to obtain $\mu = f(\text{day of year})$ and $\sigma = f(\text{day of year})$.

An alternative method was also trialled for situations when only a short historic timeseries was available (one to five years). This method reduced most of the seasonal trends in μ by using the Penman-Montheith equation, assuming cloud cover and wind speed were constant, and modelling the annual sinusoidal pattern in extra-terrestrial radiation and mean temperature. The standardisation method described above was then used for refinement. However, the benefits derived from this alternative method did not offset the added complexity.

A first order AR (1) model and an ARMA model were fitted to the Christchurch Airport ETo historic timeseries (Equation 3-2). The standardisation procedure ensures that the mean and variance of these models are equal to the mean and variance of the historic series. Model parameters, presented in Table 3-2, were manually fitted to match the correlation function of the historic timeseries (Figure 3-3). The appropriateness of a Gaussian distribution for the random variable ϵ is shown in Figure 3-4.

$$\text{eto}'_t = \Phi_1 \text{eto}'_{t-1} + \epsilon_t \quad \text{[a] AR(1) model}$$

$$\text{eto}'_t = \Phi_1 \text{eto}'_{t-1} + \Phi_2 \text{eto}'_{t-2} + \Phi_3 \text{eto}'_{t-3} + \theta \sum_{j=1}^{j=\tau} \epsilon_{t-j} + \epsilon_t \quad \text{[b] ARMA model}$$

Where:

eto'_t = Standardised daily ETo as $f(t)$ (unitless)

Φ_* , θ & τ = Model parameters (Table 3-2). Note θ is a single constant.

ϵ_t = Random variable with Gaussian distribution and mean zero as $f(t)$ (unitless)

t = Time (day of year)

Equation 3-2: AR and ARMA models used for timeseries analysis of Christchurch airport reference ET.

⁷ Forty-four years of data meant sample sizes were $365/30 \times 44 = 535$.

Table 3-2: Manually fitted AR and ARMA model parameters for Equation 3-2.

Parameter	Model	
	AR	ARMA
Φ_1	0.36	0.31
Φ_2	-	0.04
Φ_3	-	0.03
θ	-	0.015
τ	-	20

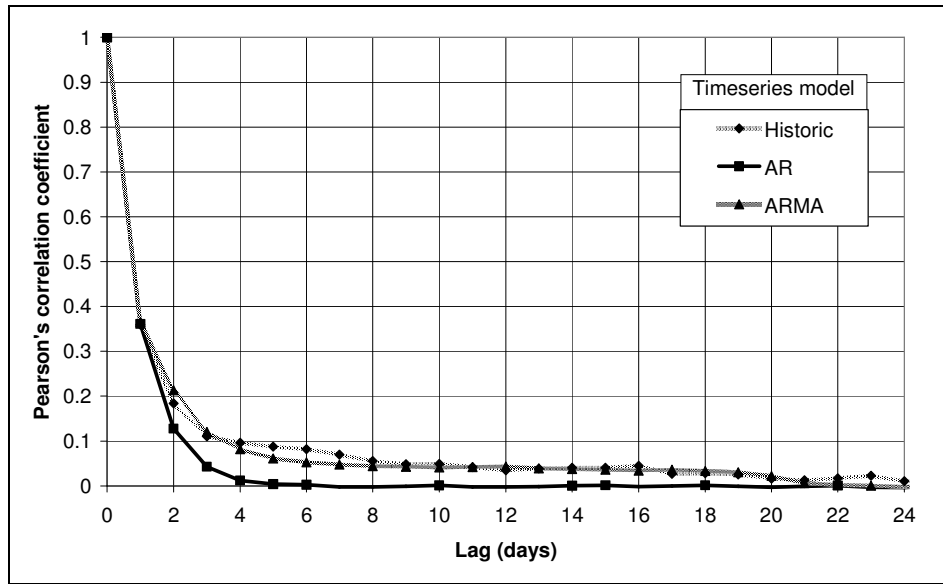


Figure 3-3: Observed and modelled correlation functions (correlation between day_i and $\text{day}_{i-\text{lag}}$) for standardised ETo, for both the AR and ARMA timeseries model (Equation 3-2).

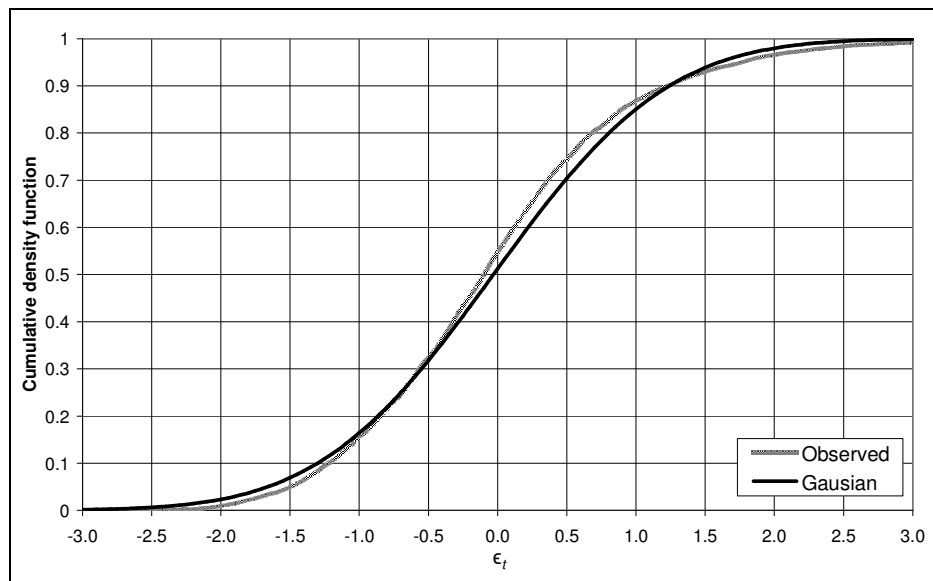


Figure 3-4: The assumed distribution of the random variable ϵ_t for the AR and ARMA models in Equation 3.2 is Gaussian, with a mean of zero and a variance of one. The observed distribution is for Christchurch Airport standardised historic ETo.

Figure 3-3 shows the ARMA accounted better for the slower decay in correlation (compared with the AR model). Correlation for a lag of one day was also plotted against the time of season (not shown) – no clear seasonal trends were observed.

Figure 3-4 shows that the assumption in Equation 3-2 of a Gaussian distribution for ϵ is very good. The distribution of ϵ was also plotted for various months of the year, with all months showing the Gaussian distribution assumption was reasonable.

In addition to the correlation function in Figure 3-3, long-term dependence was also measured by comparing the variability in total annual ETo, with the variability predicted by the AR and ARMA models. Estimates of annual variability for the AR and ARMA models were calculated by synthetically generating a large number of years of data; hence 95% confidence intervals are provided with these estimates. A confidence interval is not appropriate for the historic data when comparing with the timeseries models, since parameters for the timeseries models were derived from the historic data. In Table 3-3, the observed variability in annual ETo was greater than predicted by the AR and ARMA models. This indicates that while the short-term dependence was well modelled in the ARMA model (Figure 3-3), long-term trends and cycles were not fully accounted for. This suggests long-term trends and cycles do have a small impact on ETo.

Table 3-3: Comparison of observed and modelled long-term ETo variations.

Timeseries	Standard deviation in total annual ETo
Historic	49.1
AR model	30.8-34.9 ¹
ARMA model	41.3-46.8 ¹
(1) 95% confidence interval	

3.5 Rainfall

Timeseries and statistical analysis was used to quantify the stochastic properties of Christchurch Airport historic rainfall timeseries. This analysis was used in Section 3.6 to quantify the influence of different stochastic components on irrigation water use.

3.5.1 Factors affecting rainfall

In order for precipitation to occur, an air mass must be ascending and holding sufficient water vapour. As the air cools due to the decreasing pressure (with elevation), water vapour precipitates. The main forms of ascending motion that lead to precipitation are convection, cyclonic lifting and orographic lifting. Convection involves relatively small air parcels, with any resulting precipitation typically random and patchy. Cyclonic weather systems are characterised by the steady ascent of large masses of air, resulting in more uniform precipitation than convection. However, due to the highly variable structure of the cyclonic system, precipitation still has significant spatial variability. Orographic

precipitation is a result of uplift caused by mountains, and therefore is much more spatially specific (Sturman and Tapper 1996).

At any one spatial location, precipitation can be considered to be a series of short storm events with variable precipitation depths. Some clustering of events is likely, particularly when associated with a cyclonic system. Global-scale influences mean that precipitation characteristics can potentially vary significantly with the time of year, and even from year to year due to multi-decadal cycles and trends (McCarthy et al. 2001).

3.5.2 *Timeseries model options*

Since rainfall is an intermittent process, the main family of suitable timeseries models include point-processing modelling, product models and Markov chains (Maidment 1993). A common point-process model used for rainfall (the compound-Poisson model) was implemented. The key limitation of this model was that short-term dependence was not modelled. This limitation was partially overcome by implementing a first order two-state Markov chain model, where correlation between the occurrences of wet days was modelled. This model fitted well the correlation structure of the sequence of wet and dry days, but did not account for the correlation in rainfall depths. A common method that has been successfully shown to model both occurrence and depth correlation, is the first order multi-state Markov chain model (Harrold 2002). This last method was not implemented, since it was possible to derive relevant conclusions in Section 3.6 (for irrigation water use) by using only the compound-Poisson model, the two-state Markov chain model, and the historic timeseries. Previous optimal scheduling algorithms have modelled rainfall with a compound-Poisson model (Córdova and Bras 1979, Rhenals and Bras 1981) and with a Neyman-Scott cluster model (Ramirez and Bras 1985, Sunantara and Ramirez 1997). The latter model is essentially equivalent to a first order two-state Markov chain model.

3.5.3 *Interception and runoff*

In estimating parameters for the Christchurch Airport timeseries, it was assumed that the first 1 mm of rainfall was not effective due to interception by the crop's canopy and/or rapid evaporation from the very top layer of the soil surface. The validity of this adjustment depends on the soil moisture deficit and crop characteristics. This will be a reasonable assumption when the soil is relatively dry and ET rates are limited by water availability. However, when these conditions are not met, the ET rate (soil evaporation and plant transpiration) will decrease as the intercepted water evaporates. As such, this intercepted water does effectively contribute to soil moisture by reducing ET losses. An advantage of making this adjustment was that very small rainfall events were ignored when estimating compound-Poisson parameters. For example, for the Christchurch Airport timeseries, rainfall events less than 1 mm accounted for almost 40% of all storm events but only 4% of the total rainfall depth. Reducing the number of storm events significantly improved the assumption of independence between events.

Unless otherwise stated, all subsequent analysis assumed 1 mm has been subtracted from *every* wet day.

All subsequent discussion in this thesis assumes that runoff losses are small and can be ignored, and that rainfall infiltration is spatially uniform. Subsequent references to storm depths and wet day rain depths are synonymous with infiltrated storm and wet day depths, respectively.

3.5.4 *Compound-Poisson process modelling*

3.5.4.1 Model description

The particular compound-Poisson model used in this research assumed storm events were a Poisson process (independent and instantaneous), and that storm depths were independent and had a Gamma distribution. This model was used by Córdova and Bras (1979) in their optimal irrigation scheduling algorithm. The main limitation of this model is the assumption of independence between storm events, since it is known that some short-term correlation exists.

The Poisson distribution of n storms arriving on a given day, and the Gamma probability density function of storm depths (u) are given by Equation 3-3.

$$PD_s(n) = \frac{\lambda^n e^{-\lambda}}{n!} \text{ for } \{n = 0, 1, 2, 3, \dots, \infty\} \quad [\text{a}]$$

$$PDF_u = \frac{\alpha_s^{\beta_s} e^{-\alpha_s u} u^{\beta_s-1}}{\Gamma(\beta_s)} \quad [\text{b}]$$

$$\lambda = \frac{S}{T} \quad [\text{c}]$$

$$\alpha_s = \frac{\bar{u}_s}{\sigma_s^2} \quad [\text{d}]$$

$$\beta_s = \frac{\bar{u}_s^2}{\sigma_s^2} \quad [\text{e}]$$

Where:

$PD_s(n)$ = Probability distribution of n storms occurring within one day

PDF_u = Probability density function of storm depths u (mm)

λ = Mean storm frequency (storms/day)

α_s = Gamma scale parameter for storm depths (mm^{-1})

β_s = Gamma shape parameter for storm depths (unitless)

S = Total number of storms

T = Total number of days

\bar{u}_s = Mean storm depth (mm)

σ_s^2 = Storm depth variance

$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt = \text{Euler's Gamma function}$

Equation 3-3: Compound-Poisson rainfall model, which assumes Poisson storm arrivals and Gamma rainfall storm depth distributions.

3.5.4.2 Derivation of daily timeseries parameter estimation method

Background

The standard method that the author has encountered for parameter estimation for a compound-Poisson rainfall model, is that of Restrepo-Posada and Eagleson (1982). This method requires hourly data and the setting of a single parameter – the minimum period of time of no rain that separated two storm events. One limitation of this method is that because the assumption of a zero-storm duration is not strictly valid, when the derived parameters are used to synthetically generate a daily time-series, this daily series does not have the same storm frequency, and depth mean and variance, as the historic daily timeseries. A second limitation is the requirement of hourly data.

An alternative parameter estimation method was developed that requires only a daily rainfall series. This method has two principal advantages over the method of Restrepo-Posada and Eagleson. Firstly, only daily historic timeseries are required. This is important since the majority of long-term timeseries available in NZ (and in other parts of the world) have daily, instead of hourly, resolution. Secondly, daily timeseries produced synthetically from parameters derived from the new method have the same

storm frequency, and depth mean and variance, as the historic series from which parameters were derived.

Derivation

Storm events are not synonymous with wet day rain depths because, according to the Poisson model, storm events are instantaneous, with the possibility of more than one storm occurring in a single day. It is not possible to distinguish from a daily timeseries the difference between wet days when only one storm occurs, and wet days when two or more storms occur. However, if the assumption is made that the underlying process is Poisson, then the expected proportion of wet days that have n storms is known. From this, a relationship can be established between the proportion of wet days and the storm frequency (Equation 3-4).

From Equation 3-3[a] the probability of a day being wet is:

$$P(\text{wet day}) = 1 - P(\text{dry day}) = 1 - PD_s(n = 0) = 1 - e^{-\lambda} \quad [\text{a}]$$

Given a day is a wet day, the probability of n storm events occurring is:

$$P(n | \text{wet day}) = \frac{P(n)}{P(\text{wet day})} = \frac{PD_s(n)}{P(\text{wet day})} = \left(\frac{\lambda^n e^{-\lambda}}{n!} \right) \left(\frac{1}{1 - e^{-\lambda}} \right) = \frac{\lambda^n}{n! (e^\lambda - 1)} \quad [\text{b}]$$

Given the total number of wet days (W), the total number of storm events (S) is given by:

$$\begin{aligned} S &= W (P(n = 1 | \text{wet day}) + 2 P(n = 2 | \text{wet day}) + 3 P(n = 3 | \text{wet day}) + \dots + \infty P(n = \infty | \text{wet day})) \\ &= \frac{W \lambda \left(1 + \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{3} + \dots + \frac{\lambda^\infty}{\infty} \right)}{e^\lambda - 1} = \frac{W \lambda e^\lambda}{e^\lambda - 1} = W \lambda \left(1 + \frac{1}{e^\lambda - 1} \right) \end{aligned}$$

Substituting the relationship between S and W into Equation 3-3[c]:

$$\lambda = \frac{1}{T} W \lambda \left(1 + \frac{1}{e^\lambda - 1} \right) = \ln \left[\frac{1}{1 - \frac{W}{T}} \right] \quad [\text{c}]$$

Where the proportion of wet days (W/T) is obtained from the historic daily rainfall series.

Equation 3-4: Poisson storm frequency given the proportion of wet days.

Expressing average daily rainfall both as storm frequency times mean storm depth, and wet day frequency times mean wet day depth:

$$\lambda \bar{u}_s = \frac{W}{T} \bar{u}_w \quad [\text{a}]$$

Where:

\bar{u}_w = Mean wet day rain depth (mm)

Using Equation 3-4[a] and [c], the mean storm depth is:

$$\bar{u}_s = \frac{\bar{u}_w}{\lambda \cdot \left(1 + \frac{1}{e^{\lambda} - 1}\right)} \quad [\text{b}]$$

The probability density function (PDF_{uw}) for wet day rain depths (u_w) is:

$$\begin{aligned} \text{PDF}_{uw} = & P(n = 1|\text{wet day}) \times \text{PDF}_{n=1} + P(n = 2|\text{wet day}) \times \text{PDF}_{n=2} \\ & + P(n = 3|\text{wet day}) \times \text{PDF}_{n=3} + \dots + P(n = \infty|\text{wet day}) \times \text{PDF}_{n=\infty} \end{aligned} \quad [\text{c}]$$

Where:

PDF_n = Probability density function of daily rainfall, given n storm events occur within each day.

For the case when only one event occurs, PDF_{n=1} = GD($u_w | \alpha_s, \beta_s$). Where:

$$\begin{aligned} \text{GD}(z|x, y) &= \frac{(x^y e^{-xz}) z^{y-1}}{\Gamma(y)} \\ &= \text{Gamma distribution as } f(z) \text{ given scale parameter } x \text{ and shape parameter } y \end{aligned}$$

For the case when two or more independent events occur in a day, PDF_n will have a Gamma distribution with mean and variance equal to $n \times \bar{u}_s$ and $n \times \sigma_s^2$ respectively⁸. Therefore, using Equation 3-3 [d,e], the Gamma scale and shape parameters for this Gamma distribution are:

$$\frac{n \bar{u}_s}{n \sigma_s^2} = \frac{\bar{u}_s}{\sigma_s^2} = \alpha_s \quad [\text{d}]$$

$$\frac{(n \bar{u}_s)^2}{n \sigma_s^2} = \frac{n \bar{u}_s^2}{\sigma_s^2} = n \beta_s \quad [\text{e}]$$

Using Equation 3-4[b] and [c,d,e]:

$$\begin{aligned} \text{PDF}_{uw} &= P(n = 1|\text{wet day}) \times \text{GD}(u_w | \alpha_s, \beta_s) + P(n = 2|\text{wet day}) \times \text{GD}(u_w | \alpha_s, 2\beta_s) \\ &\quad + P(n = 3|\text{wet day}) \times \text{GD}(u_w | \alpha_s, 3\beta_s) + \dots + P(n = \infty|\text{wet day}) \times \text{GD}(u_w | \alpha_s, \infty \beta_s) \\ &= \sum_{n=1}^{\infty} \frac{\lambda^n ((\alpha_s^n \beta_s e^{-\alpha_s u_w}) u_w^{n \beta_s - 1})}{(n! (e^{\lambda} - 1)) \Gamma(n \beta_s)} \end{aligned} \quad [\text{f}]$$

⁸ The mean of the sum of random variables equals the sum of the means, and the variance of the sum of independent random variables equals the sum of the variances of these variables (Kreyszig 1993, s23.8).

The variance of u_w is given by:

$$\text{Var}(u_w) = \int_0^{\infty} (u_w - \bar{u}_w)^2 \text{PDF}_{u_w} du_w \quad [\text{g}]$$

Equation 3-5: Gamma parameters derived from daily rainfall timeseries, assuming Poisson storm arrivals and a Gamma storm depth distribution.

Equations 3-5[b], and [f,g] give a relationship between the mean and variance of daily rainfall depths (obtained from historic rainfall timeseries), and α_s and β_s . Equation [f,g] required numerical integration. A numerical solver was required to use Equations [b], and [f,g] simultaneously to solve for α_s and β_s .

The parameter estimation procedure was verified by generating a 1000 year synthetic daily rainfall series, assuming a compound-Poisson process as described above. Equation 3-4 and Equation 3-5 were able to predict exactly the input parameters λ , α_s , and β_s for the synthetic series.

3.5.4.3 Parameters

Compound-Poisson rain parameters for Christchurch Airport by month are presented in Table 3-4. From this table, it can be seen that seasonal variations in rainfall characteristics are measurable, but not great. In general, autumn and winter are wetter than spring and summer.

Table 3-4: Poisson arrival, Gamma depth distribution rainfall parameters calculated from Equation 3-4 and Equation 3-5, given Christchurch Airport daily timeseries from 1960 to 2004.

Month	Estimated parameters			Mean daily rainfall (mm/day)
	λ (storms/day)	α_s (mm ⁻¹)	β_s (unitless)	
January	0.210	0.0839	0.494	1.24
February	0.205	0.1402	0.814	1.19
March	0.224	0.0574	0.379	1.48
April	0.237	0.0744	0.468	1.49
May	0.254	0.1309	0.760	1.47
June	0.292	0.0950	0.530	1.63
July	0.291	0.0846	0.521	1.79
August	0.281	0.0845	0.473	1.57
September	0.217	0.1312	0.719	1.19
October	0.230	0.1131	0.574	1.17
November	0.266	0.1005	0.515	1.36
December	0.232	0.1257	0.618	1.14

The performance of the compound-Poisson model is tested in Section 3.5.7. Synthetically generated rainfall events using this model are used in the simulation studies in Section 3.6.

3.5.5 *Modelling correlation between rainfall occurrences*

Given the processes that generate rain (Section 3.5.1), it would be expected that some clustering of storm events will occur, particularly when rain is associated with cyclone systems. The two most common methods of modelling short-term storm arrival dependence in rainfall series are a Neyman-Scott cluster model and a first order two-state Markov chain process.

The Neyman-Scott model assumes macro-storm events arrivals are a Poisson process, and within each of these macro-storms, a random number of instantaneous micro-storm events occur according to a given probability distribution. Storm depths for macro-storms are the sum of micro-storm depths, and micro-storm depths are assumed to be independent. Such a cluster model was used by Ramirez and Bras (1985) in their optimal irrigation scheduling algorithm. They assumed that the number of micro-storms per macro-storm has a geometric distribution, and that micro-storm event rainfall depths have a Gamma distribution. These authors commented that the model fitted well the short-term correlation structure of rainfall events. However, for their case study, they found that there was no advantage in using this more complex cluster model, in place of the simpler compound-Poisson model. One reason for their lack of benefit is likely due to the high soil TAW (over 600 mm) used in their case study, which is much higher than typical Canterbury Plain soils (where generally TAW is less than 200 mm).

A first order two-state Markov chain process assumes that the probability of rain on a given day is dependent on whether or not rain occurred one day previous. Unlike a Poisson process, the time dimension is discretised to daily time units, therefore concern is given only to daily rainfall depths. Unlike the Poisson or Neyman-Scott models, multiple storm events within a day are not considered. Daily rainfall depths are assumed to be independent and are most commonly assumed to have either a Weibull or Gamma distribution. The two-state Markov chain process model is incorporated in the widely used climate-generating software, Climgen (Campbell 2006). The specifics of Climgen's rainfall component is described in more detail by Carlini et al. (2006).

Within this thesis, the two-state Markov chain process model was used in preference to the Neyman-Scott cluster model, due to its simplicity, particularly when parameterising using daily historic timeseries. A Gamma distribution was used for rainfall depths. Derived parameters for each month of the year are presented in Table 3-5.

Table 3-5: Estimated two-state Markov chain precipitation parameters for Christchurch Airport daily timeseries from 1960 to 2004.

Month	Wet day probability		Daily rainfall depth distribution		Mean daily rainfall (mm/day)
	P(wet dry day previous)	P(wet wet day previous)	α_w (mm ⁻¹) ¹	β_w (unitless) ²	
January	0.144	0.384	0.0801	0.523	1.24
February	0.148	0.348	0.1296	0.834	1.19
March	0.142	0.434	0.0554	0.408	1.48
April	0.157	0.415	0.0707	0.500	1.49
May	0.159	0.451	0.1195	0.786	1.47
June	0.193	0.431	0.0886	0.570	1.63
July	0.183	0.456	0.0789	0.561	1.79
August	0.175	0.461	0.0796	0.512	1.57
September	0.152	0.370	0.1219	0.744	1.19
October	0.159	0.386	0.1066	0.607	1.17
November	0.182	0.403	0.0944	0.552	1.36
December	0.165	0.369	0.1176	0.649	1.14
(3) α_w = Gamma distribution scale parameter for wet day rainfall depths					
(4) β_w = Gamma distribution shape parameter for wet day rainfall depths					

The performance of the two-state Markov chain model was tested in Section 3.5.7. Synthetically-generated climate utilising this model was used in the simulation studies in Section 3.6.

3.5.6 Modelling correlation between rainfall depths

The limitation of the two-state Markov chain model is the assumption of independence between wet day rain depths. Both a correlation and a higher mean rainfall depth when wet days occur consecutively were found (Section 3.5.7). A common method for accounting for this correlation is a multi-state Markov chain model, where a probability transition matrix defines not only the probability of rain occurrence conditional on whether the previous day was wet, but also the probabilistic depth of rainfall, given the depth of rainfall on the previous day (Srikanthan and Chiew 2003). This model has been shown to have superiority over alternative models of short-term correlation (Harrold 2002). This method was not implemented, since it was possible to derive relevant conclusions in Section 3.6 (relating to irrigation water use) by using only the compound-Poisson model, the two-state Markov chain model, and the historic climate.

3.5.7 Timeseries modelling performance

Mean rainfall (at various scales) are identical between the historic timeseries and the two timeseries models from Section 3.5.4 and Section 3.5.5. Model fitting ensures that daily variance is identical between the historic timeseries and the models; however, variance at other time scales (monthly, annual) and correlation at various time scales can deviate between the historic timeseries and these models, if the underlying assumptions of these models are not strictly correct. Modelled and observed rainfall depth distributions can also deviate.

3.5.7.1 Correlation between consecutive days

Pearson's correlation function is plotted for rainfall occurrence in Figure 3-5 and for rainfall depth in Figure 3-6. Interestingly, the single day dependence between rainfall occurrences (first-order Markov chain assumption) was able to fully account for the short-term correlation structure. As previously noted, one of the principal weaknesses of the compound-Poisson model is that short-term independence is not accounted for. Figure 3-6 shows that the two-state Markov chain model assumption of independence between rainfall depths is not strictly valid.

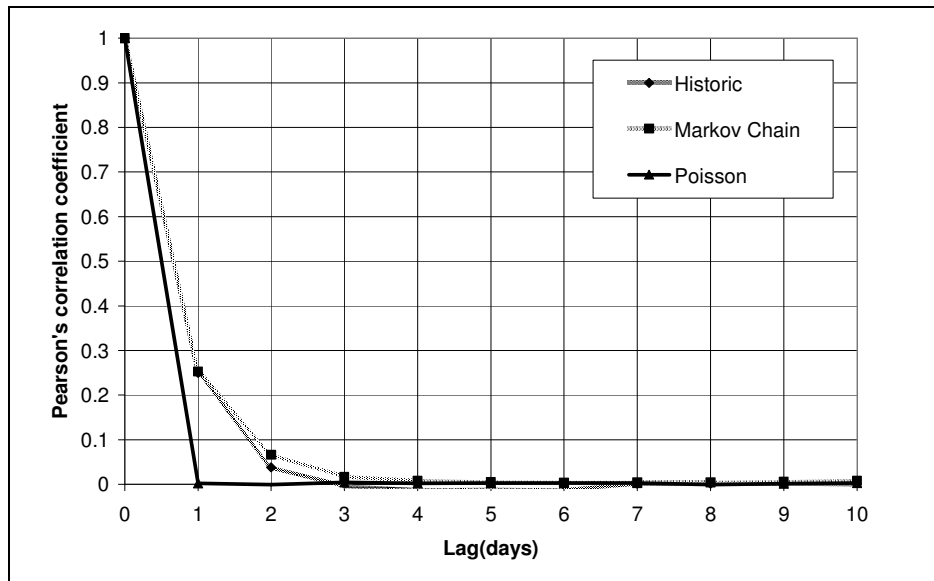


Figure 3-5: Auto-correlation (correlation between day_i and $\text{day}_{i-\text{lag}}$) between wet day events, comparing a Poisson model (Section 3.5.4), a first order Markov chain model (Section 3.5.5), and historical data.

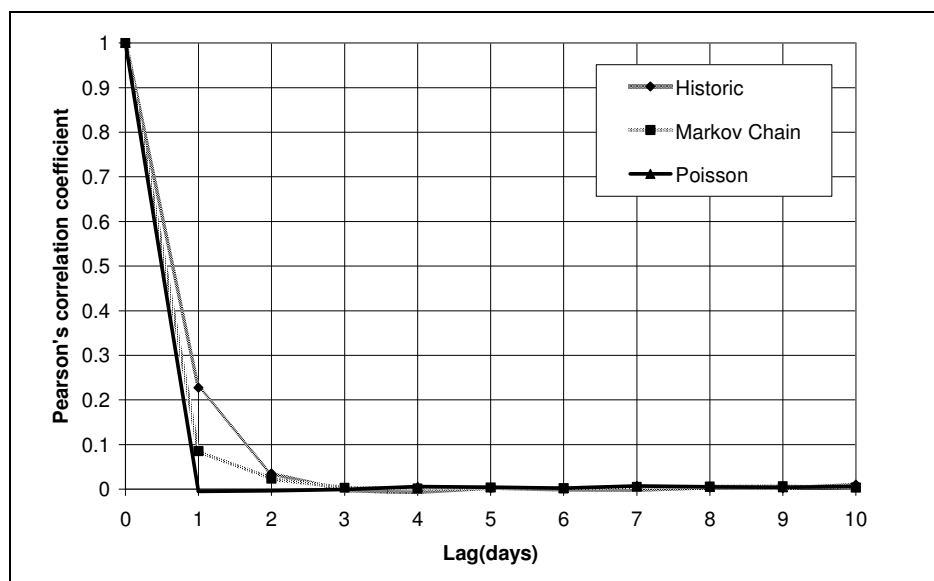
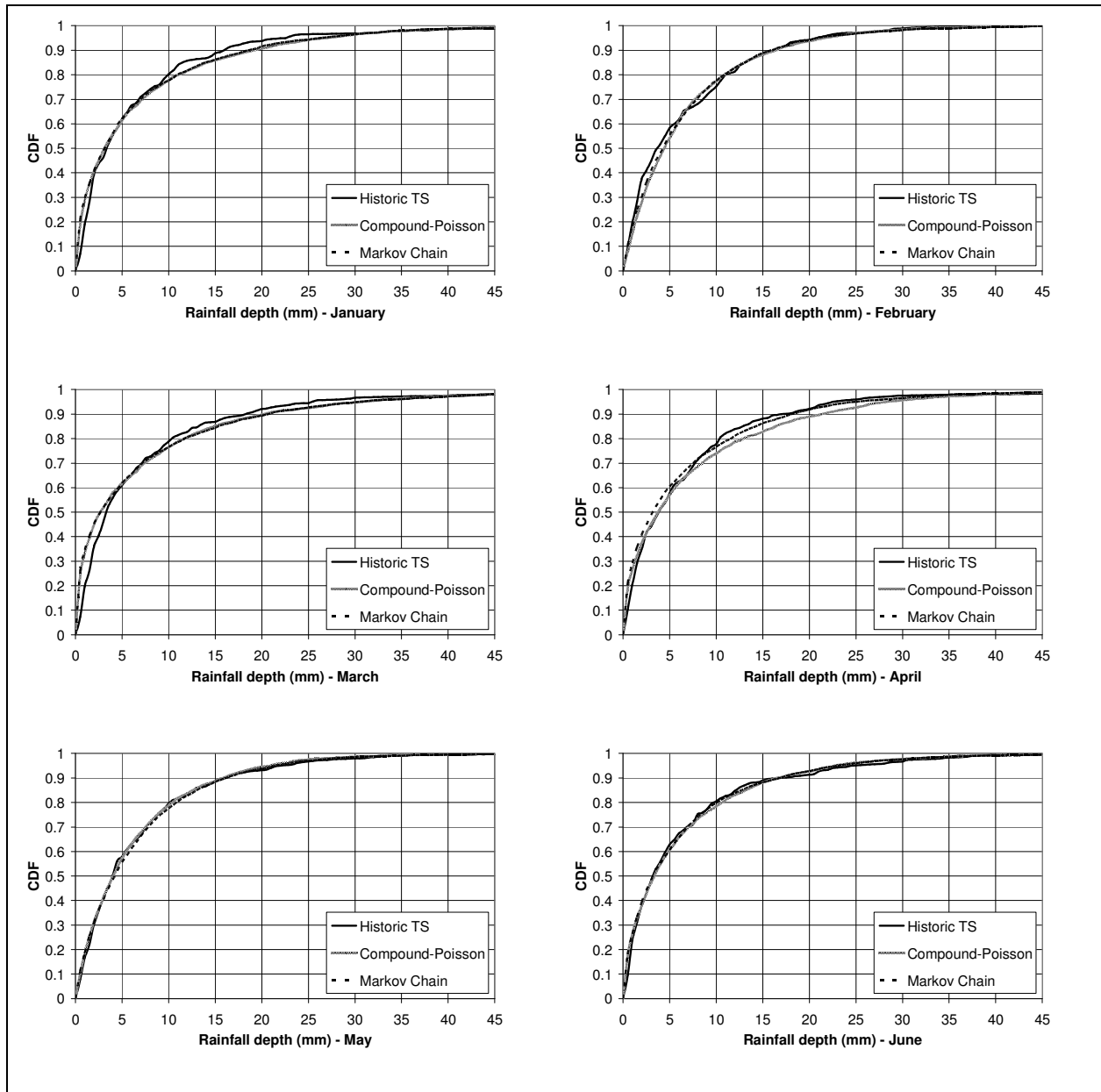


Figure 3-6: Auto-correlation (correlation between day_i and $\text{day}_{i-\text{lag}}$) between daily rainfall depths, comparing a compound-Poisson model (Section 3.5.4), a two-state Markov chain model (Section 3.3.5), and historical data.

3.5.7.2 Rainfall depth distribution

A comparison between the observed and modelled daily distribution of rainfall depths (by month) is presented in Figure 3-7. In general, the two models fit well the distribution of the observed data. Interestingly, the rainfall depth distribution was relatively constant throughout the year. Autumn and winter months are wetter due principally to a higher probability of rainfall, rather than higher rainfall depths.



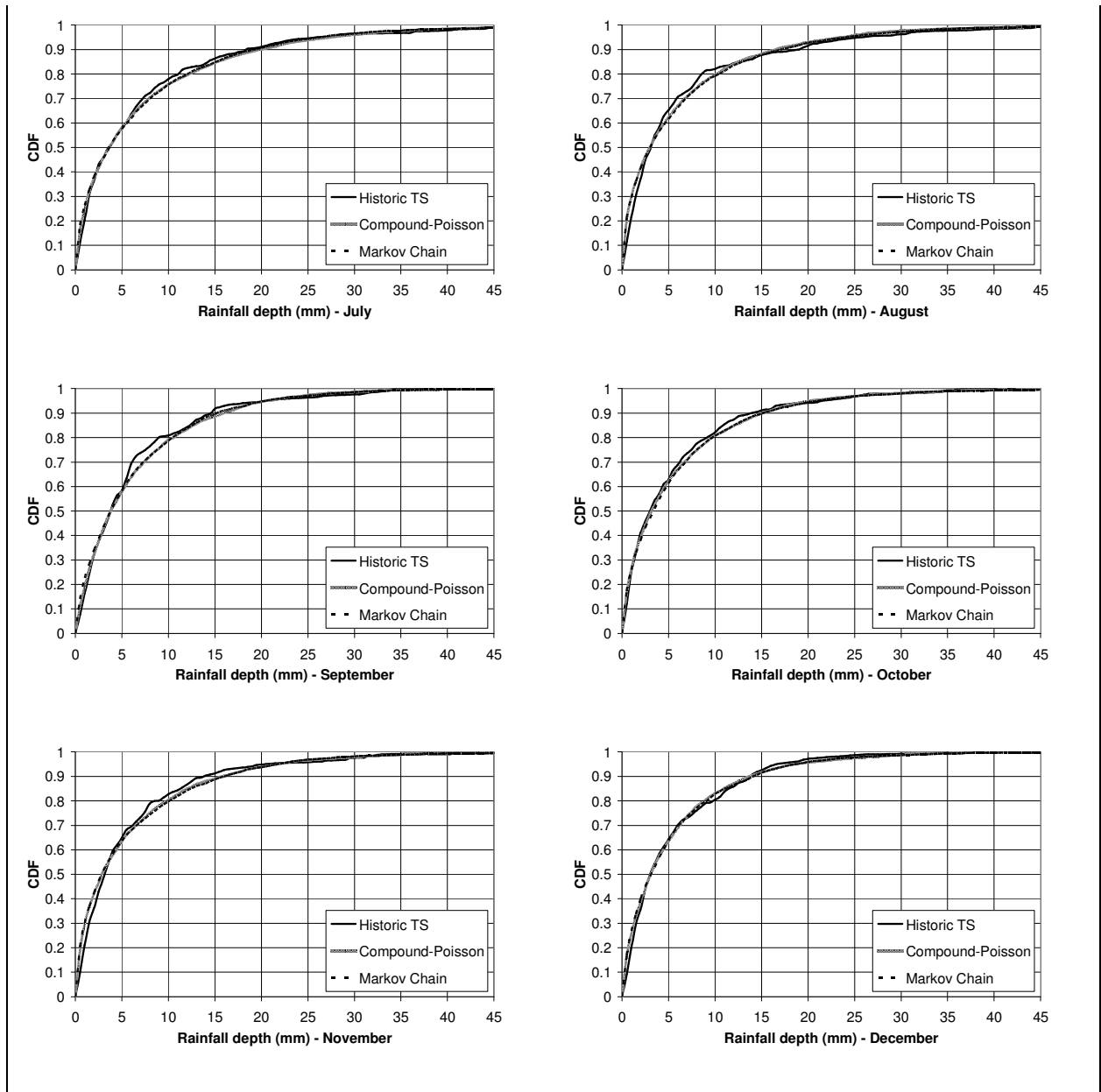


Figure 3-7: Comparison between the observed and modelled daily rainfall depth cumulative density function (CDF) for each calendar month.

The probability density function of observed and modelled daily distribution of rainfall depths (by month) were also compared, and likewise showed that in general, the two models fit well the distribution of the observed data

3.5.7.3 Long-term trends and cycles

One method for detecting long-term cycles or trends is by comparing the differences between the modelled and observed variance, at different time scales. Table 3-6 compares the expected standard deviation (calculated from synthetically-generated timeseries) for the two timeseries models with the observed standard deviation.

Table 3-6: Standard deviation in annual rainfall – modelled and observed.

Timeseries/model	SD[No. rainfall events/year] ⁽¹⁾	SD[annual rainfall] ⁽¹⁾ (mm)
Historic	10.3	133
Compound-Poisson	7.3-8.3	90-102
Markov chain	9.3-10.5	95-108
Note: (1) 95% confidence interval (CI). A CI is not appropriate for the historic timeseries when comparing with the models, since model parameters are derived from the historic series.		

In Table 3-6 part of the higher variance in the historic series can be attributed to (a) the models either totally not, or only partially, accounting for short-term correlation between rain events, and (b) long-term trends and cycles. The compound-Poisson model and the two-state Markov chain model account for 67-77% and 71-81% (respectively) of the variability in total annual rainfall. The compound-Poisson model and the two-state Markov chain model account for none, and approximately 40% (respectively) of short-term correlation in daily rainfall (Figure 3-6). Therefore it is estimated that if a multi-state Markov chain model implemented (which fully accounted for short-term correlation in daily rainfall), the model would account for approximately 77-87% of the variability in total annual rainfall⁹. Therefore, long-term trends and cycles are estimated to increase total annual rainfall variability by about 15-25%.

Historic total annual rainfall, summed over the irrigation year from June to May, is presented in Figure 3-8 for the period 1960 to 2004. The correlation function for this series is presented in Figure 3-9. The short series did not allow the identification of any significant correlation. This distribution of historic annual rainfall was also plotted (not shown) and, was found to have a Gaussian distribution.

⁹ Modelling no short term correlation accounted for 67-77% of seasonal variability. Accounting for 40% of short term correlation accounted for 71-81% of seasonal variability. Therefore accounting for all short term correlation is expected to account for about $67 + 1.0/0.4(71-67)$ to $77 + 1.0/0.4(81-77) = 77 - 87\%$ of seasonal variability.

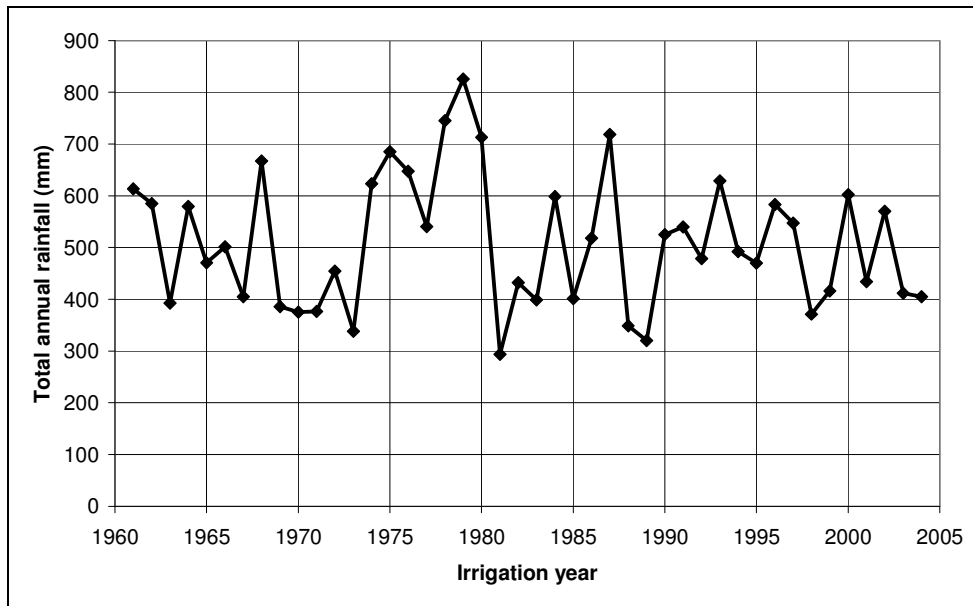


Figure 3-8: Total annual rainfall for Christchurch airport, from 1960 to 2004.

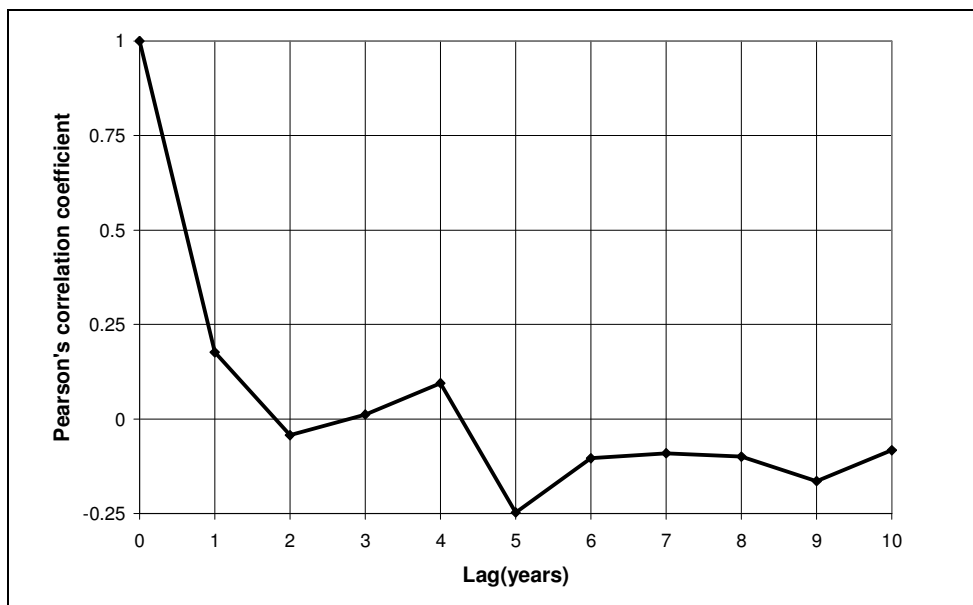


Figure 3-9: Total annual rainfall correlation function (correlation between day_i and $\text{day}_{i-\text{lag}}$) for Christchurch airport, for the period from 1960 to 2004.

3.6 Climate Model Influence on Seasonal Water Use Predictions

The impact of specific stochastic climate properties on irrigation water use was quantified by using various combinations of ET and rainfall timeseries models in farm simulations. Simulation parameters are presented in Table 3-7, and the results are presented in Table 3-8.

Table 3-7: Water balance simulation parameters: Quantifying the impact of specific stochastic climate properties on irrigation water use.

Parameter	Value
Model description	Single soil layer, single crop coefficient FAO 56 crop model after Allen et al. (1998) (Section 2.3)
Crop	Pasture – constant rooting depth and crop coefficient of 1.0
Soil TAW	80 mm
Soil Readily Available Water	40 mm
Irrigation application depth	6 mm
System capacity	6 mm/day
Irrigation season	1 September – 31 March
UCC	1.0
Trigger soil moisture level	Variable, as noted in Table 3-8
Climate data	Christchurch Airport (Table 3-1) and synthetically-generated timeseries, as noted in Table 3-8

Table 3-8: The influence of stochastic climate properties (for Christchurch) on seasonal irrigation water use. ETo and rainfall, timeseries models, and historical data (in various combinations), are used as inputs for a water use simulation.

ID	Climate timeseries		TSML ⁽¹⁾	# seasons simulated	Water use (mm/year)	
	Rainfall	ET			Mean ⁽²⁾	Std Dev ⁽²⁾
1.1	Historic	Historic	0.7	44	587	86
1.2	Historic	Historic	0.3	44	327	68
2.1	Cpd. Poisson ⁽³⁾	Mean ⁽⁴⁾	0.7	500	567-575	41-47
2.2	Cpd. Poisson	Mean	0.3	500	311-319	37-42
3.1	Mrv. Chain ⁽⁵⁾	Mean	0.7	500	573-581	45-51
3.2	Mrv. Chain	Mean	0.3	500	314-322	41-46
4.1	Mrv. Chain	AR	0.7	500	567-577	54-60
4.2	Mrv. Chain	AR	0.3	500	315-323	44-50
5.1	Mrv. Chain	ARMA	0.7	500	560-571	58-66
5.2	Mrv. Chain	ARMA	0.3	500	309-318	48-54
6.1	Mrv. Chain	Historic	0.7	528	572-580	76-82
6.2	Mrv. Chain	Historic	0.3	528	317-324	58-63
7.1	Historic	Mean	0.7	44	579	50
7.2	Historic	Mean	0.3	44	319	45
Notes: (1) Irrigation trigger soil moisture level (as a proportion of the total plant available water) (2) 95% confidence interval (since parameters are statistically estimated). Not applicable for historic data (when comparing with timeseries models), since timeseries model parameters were derived from this historical data. (3) Compound-Poisson (4) Mean values as a function of the time of year (5) First order two-state Markov chain						

The following conclusions can be drawn from Table 3-8:

- In general, there was very little difference in the predicted mean seasonal water use using different timeseries models and historic data.
- Slightly higher (1.4-2.4%) mean water use occurred due to correlation between rain events and low ET event (cf. ID 1.1/1.2 and 7.1/7.2)¹⁰.
- Clustering of rain events resulted in a minor increase in mean water use (~1%) (cf. ID 2.1/2.2 and 7.1/7.2)¹¹.
- The greatest differences between different climate models and the historic timeseries were the variability in water use. Run 3 (which neglected any correlation and ignored variability in ETo) accounted for only 50% of observed seasonal variability. Run 5 (which incorporated some short-

¹⁰ Run 1 and 7 differ only w.r.t. the ETo timeseries. Higher water use in Runs 1 indicates there is either more drainage and/or greater ET relative to Runs 7. Correlation between wet days and low ET, and between dry days and high ET, is physically reasonable and would result in both increased drainage and increased ET. $(587-579)/587=1.4\%$; $(327-319)/327=2.4\%$.

¹¹ Run 2 and 7 differ only w.r.t. the rainfall timeseries. Run 2 assumes no rainfall clusterings, while Run 7 fulling accounts for clustering. $(579-1/2(567+575))/579=1.4\%$; $(319-1/2(311+319))/319=1.3\%$.

term correlation, but neglecting any cross-correlation, or long term trends and cycles) accounted for approximately 70% of seasonal variability in water use.

3.7 Weather Forecasting

3.7.1 Background

Improving climate forecasts and greater utilisation of existing forecasting ability may enhance irrigation scheduling. From an irrigation perspective, the most useful forecast parameter is the timing and depth of precipitation. This can be a difficult parameter to predict, due to the number of factors that need to exist simultaneously to result in precipitation (Section 3.5.1).

A simple prediction method is to use historical knowledge to give the conditional probability of precipitation occurring tomorrow, given knowledge of whether precipitation occurred today. This can be effective due to the 2-3 days a depression system takes to pass over a particular location. The two-state Markov chain model implemented in Section 3.5.5 is an example of one such timeseries model.

An improvement on this simple method is to use a numerical weather prediction model. These models, which have become more common over the last two decades, provide greater prediction ability than is possible from pure statistical analysis through the incorporation of governing physical equations. In NZ, numerical modelling currently can provide estimates of timing and depth of rainfall 48 hours in advance, and probabilistic estimates of the timing of rainfall occurrences up to 15 days in advance (Renwick et al. 2006, Uddstrom et al. 2006).

In the future, increased coverage of radar measurement of rainfall will improve 48-hour predictions. Improved numerical modelling, combined with increased ability to measure the current state of the weather system through satellites in previously poorly monitored regions (e.g. Southern Ocean), will improve fortnightly probabilistic predictions (Sturman and Tapper 2006). Improved understanding of longer-scale cycles in the climate-ocean system may also enhance probabilistic estimates of total seasonal rainfall.

3.7.2 Quantification of potential water savings

Maximum potential water savings from weather forecasting were quantified by using a modified version of the SA scheduler presented in Chapter 6. The optimal scheduler was given the ability to forecast the exact future climate up to two and five days in advance. When exploring possible irrigation regimes, the scheduler has the flexibility to optimise the trigger soil moisture level for the first five days (from the current day). For the remainder of the season, the scheduler assumes a constant trigger soil moisture level. This short-term flexibility allows the scheduler to delay irrigation in response to predicted rainfall, mimicking the rationale an irrigation manager may use.

Two different farm systems were modelled to investigate how potential benefits from weather forecasting vary between different irrigation operations. Farm 1 is representative of well managed

irrigation on shallow soils, with moderate application depths. Farm 2 is representative of a well-managed irrigation of slightly heavier soils, with a low application depth. Simulation case study parameters are presented in Table 3-9, and the results are presented in Figure 3-10 and Figure 3-11. The mean annual ET is used as an indicator of total pasture production. These figures illustrate how forecasting allowed for the same amount of pasture production (expressed as annual ET) but with reduced seasonal water use. Potential water savings (expressed as a % of water use without any forecasting) is presented in these figures. For comparison, the mean annual potential ET (ET in the absence of any water stress) is 960 mm.

Table 3-9: Simulation study parameters: For quantifying weather forecasting benefits for Christchurch climate, given two different farm systems.

Parameter	Value	
	Farm 1	Farm 2
Model description	Single soil layer, single crop coefficient FAO 56 crop model after Allen et al. (1998)	
Crop	Pasture – constant rooting depth and crop coefficient of 1.0	
Soil TAW	60 mm	80 mm
Soil RAW	30 mm	40 mm
Irrigation depth	30 mm	15 mm
System capacity	8 mm/day (non-constraining)	
Irrigation season	15 September – 28 March	
UCC (Section 2.2.5)	1.0	
Trigger soil moisture levels	Optimised by scheduler, subject to total seasonal available water constraints	
Case study irrigation years	Historic climate, every odd number year, from 1961 to 2003	
Optimal scheduler, objective function sample climate years (Section 3.5)	Historic climate, every even number year, from 1960 to 2002	

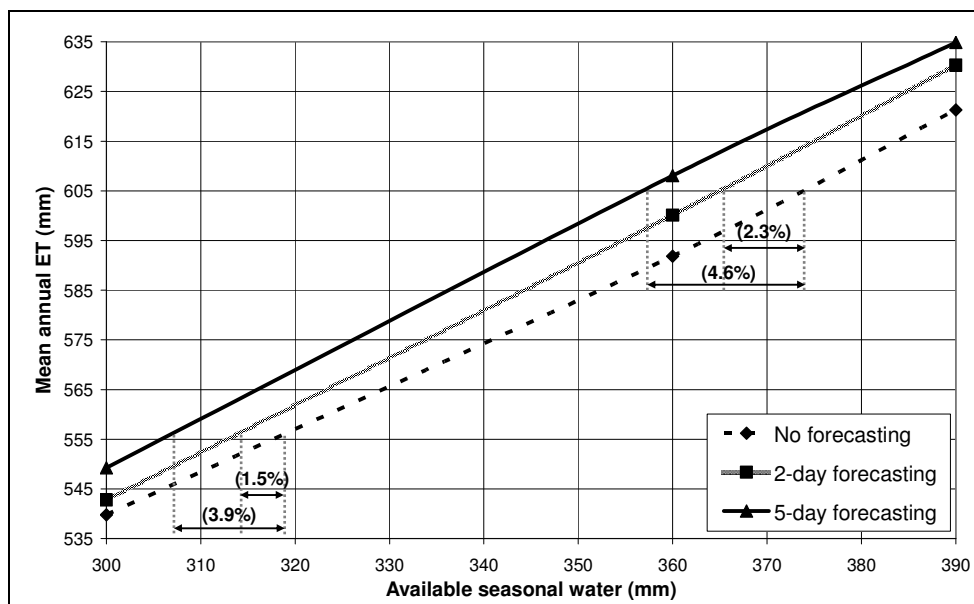


Figure 3-10: Maximum potential water savings from weather forecasting (Farm 1), for Christchurch climate. Forecasts are assumed to be 100% accurate. Farm 1 has well managed irrigation, with soil TAW of 60 mm, and an irrigation depth of 30 mm. Water savings from 2 and 5 day forecasting were 1.5-2.3% and 3.9-4.6% respectively.

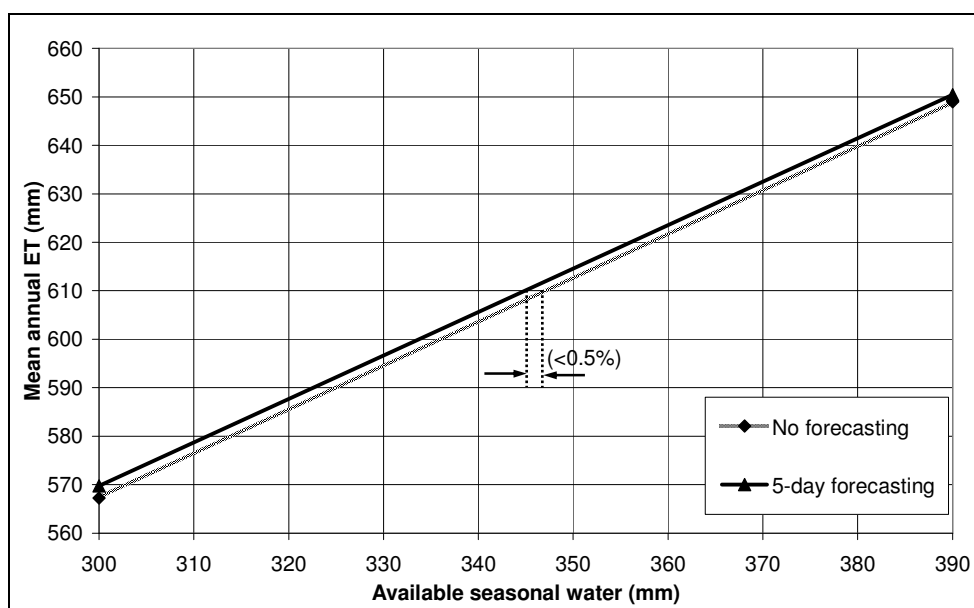


Figure 3-11: Maximum potential water savings from weather forecasting (Farm 2), for Christchurch climate. Forecasts are assumed to be 100% accurate. Farm 2 has well managed irrigation, with soil TAW of 80 mm, and an irrigation depth of 15 mm. Water savings from forecasting were less than 0.5%.

Figure 3-10 and Figure 3-11 suggest the benefits of climate forecasting will be minimal for well-managed irrigation systems with low application depths but will be more significant for irrigation systems operating on light soils, and/or higher irrigation application depths, and/or soil moisture sensitive crops. For Farm 1, the maximum possible benefit from incorporating 2-day forecasting

information was a 1.5-2.3% reduction in water use. In practice actual water saving will be less than 1.5-2.3%, since these simulation studies assumed future forecasts were 100% accurate, something which cannot be achieved in practice. Similarly for Farm 1 the maximum possible reduction in water use from incorporating 5-day forecasting information was 3.9-4.6%. For Farm 2, water reduction benefits from forecasting were negligible (less than 0.5%).

3.8 Other Effects of Climate Variability

Climate variability in other variables, particularly temperature, can have a significant influence on crop development, growth rates and disease susceptibility. Variations in the timing of crop development and growth rates do not significantly affect water use. However, for some farm operations (dairying, in particular), loss of grass production from below average soil temperatures may place greater pressure on irrigation managers to ensure that any loss in production, during the current season, from water use is minimised. Other climate influences, such as frosts and humidity variations, tend to mainly affect only certain crops.

3.9 Conclusions

Conclusions from ETo timeseries modelling were: (a) AR and ARMA (with Gaussian white noise) were both reasonable models of standardised ETo – the advantage of the ARMA was a better fit of the correlation function; (b) long-term trends and cycles increase total annual ETo variability by about 10%.

Conclusions from rainfall modelling were: (a) compound-Poisson and a two-state Markov chain process were both reasonable models of rainfall – however, short-term correlation was not accounted for by the compound-Poisson model, and only partially accounted for by the two-state Markov chain process; (b) long-term trends and cycles are estimated to increase total annual rainfall variability by 15-25%.

Conclusions from the irrigation water use farm simulation study were:

- a) In general, there was very little difference in the predicted mean seasonal water use for the different timeseries models and the historic timeseries.
- b) Slightly higher (1.4-2.4%) mean water use occurred due to correlation between rain events and low ET event.
- c) Clustering of rain events resulted in a minor increase in mean water use (~1%).
- d) The greatest differences between different climate models and the historic timeseries were the variability in water use. Different climate models accounted for between 50-70% of observed seasonal water use variability.

The compound-Poisson rainfall model, with a standardized mean ETo model, is used in the Water Use Equations in Chapter 7.

4 EXISTING COMPUTER-AIDED SCHEDULING SUPPORT

4.1 Context and Overview

In light of the constraints and modelling requirements of irrigation systems described in Chapters 1 to 3, a review of existing irrigation scheduling support tools was undertaken to identify their potential strengths and weaknesses. This background information is used in the development of the novel optimal scheduler in Chapters 6.

4.2 Single-crop Scheduling

4.2.1 *Scheduling support without computer optimisation*

The main scheduling support tools used in practice are essentially an account balance sheet for water, which keeps track of available soil moisture and allows for short-term predictions of soil moisture in response to irrigation decisions. Crop water use models are generally in the form of the FAO 56 model (Allen et al. 1998), coupled with a single or dual soil layer. Such programmes have been developed by California State University (Zoldoske 1990), Purdue University and Michigan State University (Joern et al. 1997), and the United Nations (FAO 1992), amongst others. With respect to the modelling requirements from Chapter 2, the models used in these programmes are acceptable for modelling irrigation water use. However, the programmes do not provide advice on optimal trigger soil moisture levels or advice for situations of restrictive seasonal water use limits. Prediction of farm profit is not required since optimisation is not used.

4.2.2 *Optimal scheduling using dynamic programming*

Bright (1986) identified a number of authors from the late 1960's to early 1980's, who used simplistic soil-plant-climate models, with dynamic programming optimisation, to schedule irrigation for a single crop. The most recent major development of this type of approach was the work of Bras and co-workers, who used stochastic dynamic programming (SDP) and assumed a FAO 56/FAO 33 (Doorenbos et al. 1979, Allen et al. 1998) type crop model with a single soil layer and single crop-coefficient, modelled rainfall as both a compound-Poisson process and a Neyman-Scott cluster model, and included a seasonal constraint on water use (Córdova and Bras 1979, Ramirez and Bras 1985). Modelling reference ET stochastically with a first order AR model was also investigated (Rhenals and Bras 1981). The authors were unable to show any advantage in modelling rainfall using the more complicated Newman-Scott compared with the compound-Poisson process, or in modelling reference ET stochastically rather than deterministically. One reason for this is the high soil TAW used in these authors' case studies (Section 3.4.2 and Section 3.5.5). Bright contributed to this algorithm by including an application efficiency model (Section 2.2.5). Since these works, there has been little published in this area; this is probably due to the substantial shift in research funding from irrigation quantity to water quality research since the mid 1980's, particularly in the United States.

With respect to the modelling requirements from Chapter 2, the models used in the above SDP schedulers are acceptable for modelling irrigation water use. If required, a dual layer soil can be incorporated into the SDP formulation by adding an additional state variable – an extension of existing schedulers, which all assume a single soil layer. These schedulers allow the inclusion of a seasonal water use limit. The above authors did not allow for a system capacity constraint; however, this is easily incorporated, and has been included in the SDP algorithm used in Chapter 9. Climate models used in these algorithms are acceptable (Chapter 3). The principle limitation of the SDP formulation is that it does not allow for the complex crop models required to model farm profit (Chapter 2). This is a major limitation, since the results from optimisation are only as reliable as the objective function (farm profit). Previous authors have made similar comments about the inadequacy of models within irrigation scheduler optimisation procedures (Jensen 1980, English et al. 2003).

4.3 Multi-crop Optimal Scheduling

4.3.1 *Optimal scheduling using dynamic programming*

When system capacity is not restrictive, single-crop scheduling algorithms can be extended to scheduling multiple crops using a decompositional approach. For this decompositional approach, a single-crop algorithm is run a number of times for each crop to generate a relationship between total seasonal water use and net-return. Water is then allocated seasonally between different crops (Section 2.2.4).

For the situation where system capacity is restrictive, the above simple decompositional procedure cannot be used due to the inter-seasonal dependence between irrigation schedules for different crops. The difficulty with extending standard dynamic programming to the multi-crop control problem is that the computation cost of a solution increases exponentially with the number of crops considered, since each additional crop requires at least one additional state variable. Previous authors have proposed two solutions to this problem. The first was to extend the decompositional procedure to allow for intra-seasonal constraints, and the second was to use differential dynamic programming for optimisation.

Rao and Sarma (1990) extend the decompositional procedure to schedule for multiple crops. Seasonal crop water use versus net return relationships were generated using the method described above. Water was then allocated seasonally between different crops, and a weekly schedule generated for each crop. Competition for water from various crops within a given week was then checked against system capacity constraints. Water was optimally allocated for that week, and then the schedules for each crop were separately optimised from that week until the end of the season. This process was iterative. A similar procedure was used by Sunantara and Ramirez (1997), who used the single-crop algorithm of Bras and co-workers. One limitation of Rao and Sarma's approach is that optimality cannot be guaranteed, as it can with single-crop algorithms (Section 4.2.2).

Bright (1986) proposed a method for multi-crop scheduling that used constrained differential dynamic programming. This optimisation technique has quadratic convergence (Jacobson and Mayne 1970), compared with the exponential convergence rate of standard dynamic programming. A number of important simplifications were still required to make the problem computationally feasible, and to comply with the constraints of differential dynamic programming. These included (a) assuming there is no rainfall during the stage following an irrigation decision, (b) not allowing for a fixed cost of an irrigation event, and (c) neglecting any seasonal water use limits. However, with these assumptions, Bright's method was able to guarantee optimality.

The computational powers of computers have increased several orders of magnitude since Bras and co-workers, and Bright undertook their analysis in the late 1970's and early 1980's. This increase in power has been used to extend the above single crop scheduling algorithm of Bras and co-workers to multiple crops, using standard SDP. This algorithm allows for system capacity constraints without needing to make the simplifications made by Bright. Computational demand is still a major constraint, and, in its current form, a maximum of three competing crops can be scheduled. This algorithm is presented in Chapter 9, and is used to provide a baseline comparison for the novel optimal scheduler (Chapter 6).

In addition to computational limitations, multi-crop SDP schedulers inherit the simplistic crop model limitations of single crop SDP schedulers (Section 4.2.2).

4.3.2 Optimal scheduling using a genetic algorithm

Wardlaw and Bhaktikul (2004) used a genetic algorithm (GA) to optimally schedule water to different turnouts of an open-channel community distribution system. Although their particular problem involved a distribution system, the mathematical structure of the problem is similar as for on-farm multi-crop irrigation scheduling. Their use of a heuristic method for optimising irrigation decisions, coupled with simulation (in time) as part of the objective function, provided for the potential to overcome the limitation of over-simplistic soil-plant models inherent in dynamic programming methods. Heuristic methods, including GAs are further described in Chapter 5. Their approach was to schedule water such that the soil moisture at various paddocks was maintained between field capacity and wilting point, thereby minimising either excess drainage or serious water stress. Wardlaw and Bhaktikul used the FAO 56 dual crop coefficient approach for ET as described by Allen et al. (1998), together with a multi-layer soil (Section 2.3). Simple continuity constraints were used to describe which canal branches could operate concurrently. Such constraints could have alternatively been on-farm system capacity constraints. Canal hydraulics and routing times, future rainfall, and a seasonal water limit were not considered.

Two different methods were used for coding the decision variables. The first method was termed the *0/1 approach*. For this approach, each turnout had a series of Boolean decisions of the days during the season when water would be supplied. For a season length of 100 days and nine different turnouts,

this resulted in a GA string length of 900 bits. The second coding method was termed the *Warabandi approach*, where only three decisions were required for each turnout for a season. These decisions were the start date, the duration of irrigation, and the return interval. This approach reduced the GA string length for a nine-turnout system down to 27 real value numbers. A simple GA (Goldberg 1989) with a crossover probability of 0.85 and a mutation rate of 0.05 were used for optimisation. Violation of delivery and target soil moisture constraints were dealt with using penalty functions. Wardlaw and Bhaktikul (2004) found that the 0/1 approach was able to produce an equitable allocation regime between different turnouts. The Warabandi approach was found to be inefficient as excess water was applied in the shoulders of the season when ET requirements were low. A limitation of Wardlaw and Bhaktikul's optimization method is the high proportion of strings that violate system constraints, resulting in (a) the algorithm being overly-sensitive to the weighting of individual penalty terms, and (b) inefficient optimisation.

4.4 Conclusions

The principal limitation of existing schedulers that use dynamic programming is that soil-plant models, required in order to satisfy the mathematical and computational requirements of dynamic programming, are over-simplistic. An alternative scheduling procedure by Wardlaw and Bhaktikul (2004) which used a heuristic method for optimisation of irrigation decisions and simulation in the objective function, provided the potential to incorporate more complex soil-plant models. However, when applied to optimal on-farm irrigation scheduling, this method has an inefficient optimisation procedure, does not optimise target soil moisture levels, neglects future rainfall, and does not allow for seasonal water use limits.

5 OPTIMISATION METHODS

5.1 Context and Overview

An overview of common optimisation and heuristic methods is presented in the context of their suitability or otherwise for irrigation scheduling optimisation. Examples are given for application of a particular technique to either (a) optimal irrigation scheduling, or (b) problems with a similar structure to optimal irrigation scheduling. Constraints and limitations of particular methods are viewed in light of the modelling requirements detailed in Chapter 2. Conclusions from this chapter are used in selection of a suitable optimisation method for the novel optimal scheduler in Chapter 6.

5.2 Methods Guaranteeing Optimality

5.2.1 Introduction

Formal optimisers¹² aim to guarantee an optimal solution within a specified tolerance level and problem restrictions. This is in contrast to heuristic methods, which cannot guarantee optimality but aim to provide a good solution. The most basic form of formal optimisation is an evaluation of all possible options. This method requires few assumptions about the form of the objective function; however, the computational requirements increase exponentially with increasing degrees of freedom. More efficient optimisers utilise the structure of particular types of problems. An overview of some common methods used in decision optimisation is presented below. Although gradient methods are listed in this section, they can guarantee finding only a local (which may not necessarily be the global) optimum.

5.2.2 Linear and quadratic programming

Linear programming (LP) requires a linear objective function (with continuous decision variables) and linear constraints. Problems of this structure will always have an optimal value on an extreme (or corner) point of the solution space (Winston and Venkataramanan 2003). The most widely used algorithm in LP is Dantzig's simplex method, which utilises the corner point optimality structure to obtain an optimal solution in linear time (Press et al. 2002). Linear convergence means LP is a highly efficient method that can be used whenever a problem can be structured in a linear manner. Integer programming is an extension of this method that seeks to relax the requirements for continuous decision variables by bounding an integer solution with continuous variable solutions. LP is a commonly used optimisation method. Two agricultural decision optimisation examples are (a) pre-

¹² Throughout this thesis, "optimisation" is generally used to refer to any method that seeks to find the best solution irrespective of whether or not true optimality can be guaranteed. When explicitly referring to the formal definition of optimisation, the phrase "formal optimisation" is used.

season allocation of irrigation water (Matanga and Marino 1979), and (b) optimal rotational grazing (Woodward et al. 1993).

Quadratic programming requires linear constraints and a quadratic objective function of the form:

$$f(x) = c_0 + c_{11} x_1 + c_{12} x_1^2 + c_{21} x_2 + c_{22} x_2^2 + \dots + c_{n1} x_n + c_{n2} x_n^2$$

Where:

n = the number of decision variables

c_{**} = constants

Equation 5-1: Quadratic programming objective function.

For cases where the Hessian matrix (a multi-dimensional gradient representation of a function) of the objective function is not indefinite, the function will be convex and can be solved by gradient methods. Gradient methods can generally achieve linear convergence (Section 5.2.5). LP is a sub-set of quadratic programming, and gradient methods can also be used to solve LP problems. A decision optimisation example is Wardlaw and Barnes' (1999) application of quadratic programming to real-time water allocation from an irrigation distribution system.

5.2.3 *Dynamic programming*

Dynamic programming (DP), developed by Bellman (1957), has little in common with linear or quadratic programming. This method is useful when a problem has discrete values and can be partially decomposed with respect to time. This method breaks a large optimisation problem down into a series of smaller single time-step sub-problems. As the number of time-dependant variables increases, the computation demand increases exponentially. DP has been used by a number of authors for irrigation scheduling optimisation. Examples include the single crop scheduling algorithm by Córdova and Bras (1979) and differential DP multi-crop scheduling algorithm by Bright (2006).

5.2.4 *Bounding optimisation*

Two bounding methods are briefly described: integer programming and Lipschitz optimisation. Integer programming is a bounding method where the lower bound (where the true optimal cannot be better than the lower bound) is found by relaxing the continuity requirement and solving a pure LP problem. Lipschitz optimisation is a bounding method where the lower bound to the objective function surface (where optimal = minima) is successively refined until the desired level of accuracy is achieved. This method requires an estimate of the maximum slope of the objective function. In two dimensions, this algorithm can be visualised as building a 'saw tooth' lower bound under the objective function. The lowest 'tooth' from the previous iteration is evaluated, thus increasing the lower bound. At higher dimensions, it becomes increasingly difficult to calculate the new extreme points of the bounding surface. Liu (2003) applied this method to optimising management decisions for a dairy farm simulation.

5.2.5 Gradient methods

Gradient methods are suitable for any convex objective function. This includes all linear and many quadratic functions. These methods use estimates of local gradients to traverse in the direction of steepest descent. Two commonly used methods are the Nelder-Mead Downhill Simplex method and quasi-Newton methods.

A simplex is the most elementary geometrical figure that can be formed in N dimensions, and has N+1 sides. The Nelder-Mead Downhill Simplex method (unrelated to Dantzig's LP simplex method) "rolls a simplex downhill" until the minimum (where optimum=minimum) is contained within the simplex. The simplex is then contracted around the minimum. This method is robust and simple, and does not require direct gradient estimates (Press et al. 2002).

The Newton method is an iterative method for finding the minima (where optimum=minimum) of non-linear functions that works by approximating the function at each iteration with a parabola. In two dimensions, the algorithm has the form:

$$x_{new} = x_{old} - \frac{f'(x_{old})}{f''(x_{old})}$$

Equation 5-2: Newton's method of minimisation in two dimensions.

When $f(x)$ is a parabola, the exact minima of the objective function can be found in one iteration. The Hessian matrix is used to encapsulate gradient information at higher dimensions.

Quasi-Newton methods are based on the Newton method. However Quasi-Newton methods do not use the actual Hessian matrix for gradients, but instead use an approximation of it. A widely used quasi-Newton method is the David-Fletcher-Powell method which generally achieves linear convergence (Press et al. 2002).

Gradient methods usually converge rapidly. Optimality can be guaranteed for uni-modal functions. For multi-modal functions, gradient methods are sometimes still used in conjunction with heuristic methods. Heuristic methods are generally used to find promising region of the solution space, followed by gradient methods, which then find local optimums. Hart et al. (1998) used gradient methods and a hybrid between a genetic algorithm and the David-Fletcher-Powell method to optimise management decisions for a dairy farm.

5.2.6 Branch and bound methods

Branch and bound methods (when applied to sequential decision optimisation) can reduce the number of enumerations of decision sequences that require evaluating. This method breaks a problem into a tree structure. The 'tree' starts from a single point that corresponds to the initial conditions. From this point, it breaks into branches corresponding to the first decision. Subsequent decisions divide the

‘tree’ into more and more sub-branches. This method has two advantages. Firstly, it reduces the total amount of simulation time, since solutions (which have an initial sequence of decisions that are identical) need only perform this computation once. Secondly, ‘branches’ that are known to be poor performers (even before the entire decision sequence for the season is evaluated) can be ‘pruned’, reducing the need to evaluate all the sub-branches. Dimensionality can also be removed by restricting the range of decision sequences using prior knowledge about what decisions are likely to be optimal (Nemhauser and Wolsey 1988). This method is useful for discrete decision variables, but not for when decision variables are continuous. Woodward et al. (1995) used a branch and bound method to optimise rotational grazing.

5.3 Heuristic Methods

5.3.1 Introduction

Heuristic methods are a family of non-formal optimisation estimation techniques. In general no attempts are made to show, by either mathematical principles or evaluation of a solution space, that the heuristic technique yields *the* optimal solution. They can be applied to problems with little tractable structure. Heuristic methods have been widely used over the last two decades and have been successfully applied to a myriad of problems that have not been solved by traditional formal optimisation. The main established techniques currently used are genetic algorithms, simulated annealing, and tabu search. Neural networks are also used as optimisers, but in conjunction with one of these main heuristic methods (Winston and Venkataramanan 2003).

A major difficulty with optimising non-convex problems is avoiding being trapped by local optima. All heuristic methods have mechanisms for favouring good solutions, while allowing exploration of the search-space beyond the bounds of localised depressions. Genetic algorithms are based on the theory of neo-Darwinian evolution. They use two mechanisms – termed *cross-over* and *mutation* – to explore the solution space, while selective pressure is used to favour good solutions. Simulated annealing is based on the process of metal cooled slowly enough so that atoms are arranged in a near minimum energy state. In continuous space this method is related to gradient methods, with the variation that uphill movement in random directions are sometimes allowed. Tabu (forbidden) search is a deterministic method (unlike genetic algorithms and simulated annealing which are both probabilistic) that uses short-term memory to prevent the reversal of recent moves (cycling prevention), and longer-term frequency memory to ensure diverse exploration of the solution space.

Table 5-1 presents the number of papers within the Compendex database (Elsevier 2007) which contain the particular heuristic name and the word ‘optimisation’ in any of the fields. This provides an indication of the extent of heuristic methods in engineering literature and in particular the prominence of genetic algorithms. Results for the year 1994 and 2004 are presented.

Table 5-1: Heuristic method frequency within engineering publications illustrating the increased use of these methods for optimisation and the particular prominence of genetic algorithms.

Method	Number of papers in 1994	Number of papers in 2004
Genetic algorithm	557	1830
Simulated annealing	299	442
Tabu search	34	117

Heuristic methods are part of a larger field termed *soft computing*. The goal of soft computing is to exploit the tolerance of imprecision and uncertainty in real-world problems to achieve robustness and reduce computational demand. Other key components of soft computing are neural networks and fuzzy logic.

Heuristic methods are not a family of techniques that can be randomly applied to any problem, and produce a solution that is an improvement on random search methods. For instance, the ‘no free lunch theorem’ states that no heuristic is better than any other method (including random search) when averaged over all possible functions (Wolpert and Macready 1997). The situation is not as depressing as this theorem initially suggests when optimising many real-life functions, since structure in these functions can often be utilised. However, this does suggest that in order to use heuristic methods to their full advantage, some understanding of the structure of the problem and of the different heuristic operators is required. Generally, certain heuristic methods will favour optimisation problems with certain structures or characteristics.

An overview of the basic behaviour of genetic algorithms, simulated annealing, tabu search and neural networks is presented in the following sections.

5.3.2 Genetic algorithms

Genetic algorithms (GA), first proposed by Holland (1975), is a population-based heuristic method. They are the most widely used heuristic and have been successfully applied to a wide variety of difficult optimisation problems (Table 5-1). The initial focus of Holland’s work was not on optimisation, but the adaptation of a system to changes in the surrounding environment. The application of GAs to optimisation problems was first popularised by one of Holland’s students, in the book entitled ‘*Genetic algorithms in search, optimisation and machine learning*’ (Goldberg 1989).

GAs are modelled on the theory of neo-Darwinian evolution. Using this analogy, a population of individuals is manipulated by breeding (cross-over), random mutations, and survival of the fittest (selective pressure) for several generations, till the entire population converges to some superior solution. The widely publicised link between GAs and Darwinian evolution is likely one of the reasons for its popularity. However this link is not universally accepted. In discussing the misguided adherence to GAs based on non-scientific links with Darwinism evolution, Reeves and Rowe (2003)

write that *"This (public prominence of GAs) disparity must surely be at least partly due to the seductive power of neo-Darwinism"*. Later, the authors write that *"...hand-waving references to neo-Darwinian evolution are insufficient to justify the use of GAs as a tool for optimisation. A more limited but better analogy would be to the use of artificial selection in plant and animal breeding experiments. Here we have centuries of experience as evidence of the capacity of natural organisms to change in response to selective pressure imposed by some prior and external notion of fitness."*

The unspoken assumption of some magical force that makes Darwinian evolution possible has meant that some of the concepts in GAs are still popularised despite being proven to be false. In particular, 'implicit parallelism' based on the schema theory proposed by Goldberg (1989) has since been discredited (Reeves and Rowe 2003). GAs are not the most suitable method for every optimisation problem, as their popularity might initially suggest. They are principally suited to problems with discrete solution spaces. They can be expected to perform well for problems where there are several independent decision variables. They also have the advantage that the search of the solution space is both diverse (searches extend well beyond neighbouring points) and random, increasing robustness. However, this robustness may come at the expense of efficiency. Hybrids may result in considerable improvements in efficiency, particularly for utilising local gradients for continuous variable problems (Section 5.3.6).

The three main operators of a GA are recombination (cross-over), random neighbourhood search (mutation), and favouring of better solutions in a population (selective pressure). Of these, cross-over is unique to GA. Other heuristic methods have operators that form similar functions to mutation and selective pressure operators.

The cross-over operator exploits the possibility that there will be certain decision variables that will have an optimal value independently of other decisions, for most good solutions. These decisions are non-contentious. Therefore, these variables can be fixed and exploration focused on the remaining contentious decision variables. Cross-over works in conjunction with selective pressure (favouring of better solutions). Selective pressure results in a higher degree of probability that information from good solutions will be transferred to subsequent generations. Therefore, over a period of several generations, where there is a decision variable that is common to all good solutions, this decision variable (allele) will permeate through the entire population. Using the biological analogy, there is a loss of genetic information where a gene with a particular characteristic dominates an entire population. All subsequent generations will therefore have this same genetic characteristic.

The mutation operator is used to explore local neighbourhoods and to prevent premature loss of information from the cross-over operator. A local neighbourhood is defined as a solution that can be reached by changing only one or two decision variables, while all other decision variables are held constant. Mutations have the effect of delaying convergence to local optimum and are most important

during early populations, to ensure a diverse exploration of the solution space. During latter populations, the rate of mutation is commonly reduced to promote convergence.

Current theoretical work can provide some guidance on how best to design a GA (Reeves 2003, Reeves and Rowe 2003). GA design should first be guided by an understanding of how GAs work, and secondly by empirical performance of GAs when applied to other problems.

Wardlaw and Bhaktikul (2004) used a genetic algorithm for scheduling water from a canal system (Section 4.3.2). Barion et al. (1999) used a genetic algorithm for optimising pasture rotations (Section 6.2.1). Both Mayer et al. (1996) and Hart et al. (1998) used genetic algorithms to optimise management decisions for a dairy farm simulation (Section 6.2.2).

5.3.3 *Simulated annealing*

Simulated annealing (SA) is a local search heuristic method. SA is based on the analogy of metal cooled slowly enough so that atoms are arranged in a near minimum energy state (where minimum=optimum). In continuous space this method is closely related to gradient methods, with the variation that uphill movement in random directions are sometimes allowed. Using the analogy from metallurgy, gradient methods correspond to rapid cooling or quenching of a metal. In contrast, SA uses *slow* cooling to gradually converge to a solution.

For a metal at a given temperature, the system has an energy distributed proportional to the Boltzmann factor, which has the form:

$$BF(e) = \text{Exp}\left(\frac{-e}{k_b \text{temp}}\right)$$

Where:

$BF(e)$ = Boltzmann factor, describing the non-normalised energy distribution

e = System energy

k_b = Boltzmann's constant

temp = Temperature

Equation 5-3: Boltzmann factor as a function of temperature: The energy distribution of atoms within a metal is proportional to the Boltzmann factor.

Even at low temperature, there is a small chance of the system changing to a higher energy state. Therefore, there is a chance for the system to get out of a local energy minima in favour of finding a better, more global one. Alternatively put, the system sometimes goes uphill (poorer solution) as well as downhill, but uphill movements occur far less frequently at low temperatures.

SA can theoretically be shown to guarantee convergence to a global minima if a sufficiently slow enough cooling temperature is used (Henderson et al. 2003). It intuitively makes sense that SA could be a useful global optimiser. At high temperatures, the search focuses on finding large depressions,

with any small depressions easily skipped over. As the temperature cools, increasing attention is given to small local depressions.

In common discrete space SA algorithms, an evaluation point is randomly chosen in the neighbourhood of the previous evaluation point. Upon evaluation, the point is always accepted if it is a downhill movement, and has a Boltzmann probability of being accepted if it is in an uphill direction (Press et al. 2002, S10.9). This SA is inefficient for problems with continuous variables, since no advantage is taken of the natural smoothness in local topography. Furthermore, in narrow valleys, the traditional SA algorithm will almost always propose the evaluation of an uphill point even when a local downhill move exists. In response to this inefficiency, Press and co-workers proposed a continuous space algorithm that is a modification of the Downhill Simplex method (Section 5.2.5). Instead of the deterministic value for each vertex of the simplex, a logarithmically distributed random variable – proportional to the temperature (Equation 5-3) – is added to existing vertexes, and a similar random variable is subtracted from the proposed new vertex point. The result is that occasionally the vertex will move in an uphill direction.

Compared with GAs, SA is easier to implement, with only two significant parameters that requires setting: the initial temperature and the rate of cooling. Extended and random exploration of the solution space makes the method relatively robust. Press and co-workers' SA algorithm for continuous variables is well suited to problems with continuous variables and boundary conditions. An extension to this method presented in Chapter 8 allows for an efficient incorporation of equality constraints. Mayer et al. (1996) used SA to optimise management decisions for a dairy farm simulation (Section 6.2.2).

5.3.4 *Tabu search*

Tabu search is a local search heuristic method. Similar in some aspects to SA, this method has a single 'rover' that moves through the solution space in simple steps. Each iterative move is to a neighbouring point, where a neighbourhood is defined as all points surrounding a particular point that can be reached by changing just one variable. Tabu search methods incorporate short- and long-term memory, which seek to prevent cycling and provide diverse coverage of the solution space, respectively. The methodology is as follows: (a) A single rover starts at some arbitrary position on the solution space; (b) The rover evaluates all neighbourhood points and moves in the direction of steepest ascent – this process continues until the rover encounters a local minima; (c) Upon reaching a local minima, the rover attempts to climb out by moving in the direction of shallowest gradient. Back-tracking by the way the rover entered the local minima is avoided by keeping a list of recent moves and forbidding reversal of these moves. A second tool termed *long-term memory* is used to help ensure over a long period of time diverse coverage of the solution space is achieved. Long-term memory is a record of the number of times that a particular decision variable has been changed. When

the rover is in a local minima, moves favour changing a decision variable that is seldom changed (explored) (Gendreau 2003).

Unlike most other heuristic methods, tabu search is generally fully deterministic. This increases the efficiency for a well-designed problem specific algorithm, but reduces the robustness of any one algorithm performing over a variety of problems. Local gradients are utilised. Tabu search are often used in combination with other heuristic methods (Gendreau 2003).

Mayer et al. (1998b) applied tabu search to a farm management optimisation problem, which used continuous decision variables. The authors found that at higher dimensions tabu search was impractical due to the large Tabu list needed to escape local minima.

5.3.5 *Neural networks*

Artificial neural networks are based on analogy of the vertebrate brain structure, and are one of the cornerstones of soft computing. They are used widely for pattern recognition, forecasting, control, content-addressable memory, and optimisation. They are able to handle imprecise and probabilistic information, and to learn and generalise from known tasks or examples to unknown ones (but only within the decision space since while interpolation is generally good, extrapolation is often poor). By themselves, artificial neural networks are not well structured for optimization; however, they are often combined with other heuristic methods to produce optimal control algorithms. Typically, an artificial neural network would be responsible for modelling the system, and a heuristic (principally GA or SA) would adjust the weights of the neurons to optimise the system (Potvin and Smith 2003).

Morimoto et al. (1997) used a genetic algorithm to optimise a neural network model of fruit storage. The real-time optimal controller used a neural network to ‘learn’ about the response of fruit to different temperatures and humidity. This model of the system was then optimised using a GA to predict the best temperature and humidity management regime.

5.3.6 *Meta-heuristic methods*

A myriad of heuristic variants and hybrids have been proposed in the last few years. Hybrid heuristic methods may be suitable where a particular problem has characteristics that favour more than one type of heuristic, particularly hybrids between local search methods and population-based methods (e.g. Section 6.2.2, Hart et al. 1998).

5.4 Conclusions

Common optimisation methods that can (under certain circumstances) guarantee an optimal solution include Linear and Quadratic Programming, dynamic programming, bounding methods, gradient methods, and branch and bound methods. The principle limitation of Linear, Quadratic and dynamic programming, and branch and bound methods, is their likely incapability to incorporate the complex models required for predicting farm profit (Chapter 2). Previous optimal irrigation schedulers have been limited by over-simplistic farm models (Chapter 4). Bounding and gradient methods have been used for optimising management decisions for farm simulations that had little tractable structure; however, for these problems, neither method could guarantee optimality (Sections 6.2.2). In the case of bounding optimisation, it was not possible to know for certain maximum solution space gradients. For the gradient method, the solution space was uni-modal, resulting in the local optimum (found by this method) not necessarily corresponding to the global optimum.

Heuristic methods are a family of optimisation techniques that can be applied to problems with little tractable structure, making them suitable for optimising management decisions in a complex farm system simulation. Generally no attempts are made to show, by either mathematical principles or evaluation of a solution space, that the heuristic technique yields *the* optimal solution. Heuristic methods, compared to bounding and the traditional gradient methods, generally have an advantage of being more robust for complex problems.

The main heuristic methods currently used are GA, SA and Tabu search. GA can be expected to perform well for problems where there are several independent decision variables. Search of the solution space is both diverse (searches extend well beyond neighbouring points) and random, increasing robustness. However, this robustness can come at the expense of efficiency. SA is easier to implement than GA or tabu search, and has only two main parameters to set. Extended and random exploration of the solution space makes the method relatively robust. Significant use of local gradients is made, particularly at low temperatures. This can result in fast convergence for continuous variable low dimensionality problems. Tabu search is more difficult to implement than SA. It is deterministic, increasing efficiency for a well-designed problem-specific algorithm, but reducing the robustness of any one algorithm applied to a range of problems. The method may not be suitable for continuous variable problems at higher dimensions. It is often used in conjunction with other heuristic methods. By themselves, artificial neural networks are not well structured for optimisation; however, they are often combined with other heuristic methods to produce optimal control algorithms. Hybrid heuristic methods may be suitable where a particular problem has characteristics that favour more than one type of heuristic.

No one heuristic method stands out as the best method for every optimisation problem. For low dimensionality optimisation problems, ‘off the shelf’ heuristic methods may be applied. However, as the dimensionality of the problem increases, it is more likely that a custom designed heuristic –

designed to fit the structure of the objective function and constraints – will be required. In this situation, a qualitative understanding of the mechanisms that underlie certain heuristic methods is more important than comparing the performance of various methods from literature, since the objective function being optimised in literature may be quite different to the objective function at hand. Simple methods based on first principles are desirable, since complex methods are more difficult to understand, parameterise and implement. Selection and design of an appropriate heuristic for the SA scheduler is discussed further in Chapters 6 and 8.

6 OPTIMISATION OF IRRIGATION SCHEDULING USING A HEURISTIC METHOD

6.1 Context and Overview

This chapter presents a novel solution for optimal irrigation scheduling, that meets the objective function modelling requirements for farm profit (as calculated by a complex farm system simulated over several years of climate data), and constraint requirements of limits on daily and seasonal water use (Chapters 1 to 3). This novel scheduler (the SA scheduler) draws on the experience of previous authors' optimal irrigation scheduling methods (Chapter 4), while considering alternative optimisation techniques (Chapter 5), to overcome weaknesses of previous schedulers. Experience is also drawn from two optimisation problems that are closely related to irrigation scheduling: rotational grazing and optimum farm management (Section 6.2). Two specific novel components of the SA scheduler – (a) a relationship between irrigation management and mean water use, and (b) a custom heuristic method – are developed separately in Chapters 7 and 8 (respectively). The SA scheduler's performance is demonstrated in two sets of case studies. The first set of case studies compares the SA scheduler's performance with known optimum solutions (Chapter 9), while the second set of case studies (Chapter 10) demonstrates the ability of the scheduler to incorporate complex farm system models.

6.2 Related Optimisation Problems

Two optimisation problems – rotational grazing and optimum farm management – are closely related to optimal irrigation scheduling. Previous approaches to these two problems are presented, with the experience from these solutions used in the development of the SA scheduler.

6.2.1 *Optimal rotational grazing*

Optimal rotation grazing has a similar mathematical structure to optimal irrigation scheduling. Grazing management involves daily decisions during the season as to which paddocks should be grazed. This parallels the daily decisions in irrigation scheduling as to which paddocks should be irrigated. Other similarities include a farm model as part of the objective function, an inability (generally) to decompose the optimisation problem spatially or temporally, and a reservoir of the resource being allocated (herbage mass for grazing, soil moisture for irrigation) that buffers the system. Woodward et al. (1995) and Barioni et al. (1999) have both investigated optimal rotation grazing.

Woodward et al. (1995) considered the problem of optimal rotational grazing on a dairy farm during the critical period of mid July to mid September, when feed is in limited supply. The authors used a simple pasture model, where growth is only a function of the herbage mass. Decision variables were related to specific events (as opposed to general management decisions). They initially considered allowing paddocks to be grazed in any sequence. However, evaluating all possible decision sequences

for twenty Paddocks over a 60-day period would have required 20^{60} evaluations of the objective function. By discarding known sub-optimal strategies prior to beginning the search, they were able to acceptably reduce the dimensionality of the problem by multiple orders of magnitude, so that it could be solved using branch and bound optimisation.

Barioni et al. (1999) considered the problem of optimal rotational grazing on a sheep farm over a 12-month period. Their farm model was considerably more complex than that used by Woodward et al. (1995), with a larger number of decision and state variables. They used a GA for optimisation, citing that the multi-dimensional nature of the grazing decision problem makes it almost impossible to determine an optimum decision variable sequence based exclusively on a series of sensitivity analyses or linear search of each variable in turn. Furthermore, they considered other mathematical programming techniques computationally impractical. In contrast to the approach by Woodward et al., the decision variables were general management decisions, such as the rotation length for each month, rather than an exploration of all possible Paddock sequences. Optimising general management decisions reduced the optimisation problem dimensionality, allowing a number of other management aspects to be optimised simultaneously, such as the use of nitrogen fertiliser.

6.2.2 *Optimal farm management*

When decision variables describe general management decisions rather than individual events, irrigation scheduling and rotational grazing scheduling are a subset of the problem class of optimisation of general farm management decisions. For example, Barioni et al.'s (1999) optimisation decision variables described general principles of how to manage rotational grazing, in contrast to Woodward et al.'s (1995) decision variables, which described specific events.

Historically, a large amount of research has been undertaken in the optimisation of agricultural systems. The majority of these studies have used mathematical programming for optimisation. However, many real-life problems cannot be accurately modelled within the constraints of mathematical programming. Alternatively, they may have too many dimensions such that analytical techniques are currently computationally infeasible. As a result, there is a growing use of heuristic methods for system optimisation (Mayer et al. 1996).

Mayer et al. (1996, 1998a, 1998b, 1999a, 1999b) used gradient and heuristic methods for optimising a dairy farm simulation with up to 40 management decision variables. These authors found that the Downhill Simplex method of Nelder and Mead and quasi-Newton type methods performed poorly, since they converged to local rather than the global optimum. They investigated four heuristic methods: SA, GA, evolutionary strategies and tabu search. The tabu search performed poorly, with the authors commenting that this method was not well suited to problems with high-dimensionality and continuous variables (Mayer et al. 1998b). SA tended to out-perform GAs and evolutionary strategies at low dimensionality (<20 decision variables), but was out-performed by these methods as dimensionality increased. The SA algorithm used was that of Ingber (1996). This algorithm

discretised the solution space despite the original decision variables being continuous. A more efficient algorithm for this problem may be the SA method for continuous variables by Press et al. (2002). The authors found GAs tended to out-perform evolutionary strategies.

Liu (2003) trialled both a Simple GA, after Goldberg (1989), and Lipschitz optimisation for optimising management decisions in Dexcel's dairy farm simulation (Sherlock et al. 1997, McCall and Bishop-Hurley 2003). Unsustainable farm management practices were penalised by including a penalty term in the objective function. Liu found that Lipschitz optimisation was out-performed by the Simple GA as the dimensionality of the problem was increased. The author commented that the Simple GA worked best with a population size of only 10, real-value coding, non-zero selection pressure, retention of a large portion of the population (semi-steady state approach), and low mutation rates.

Hart et al. (1998) used GAs, gradient methods and hybrids of these two methods to optimise management decisions in the UDDER dairy farm model by Larcombe (1998). Management decision variables included the rotational grazing lengths, dates for calving dry-off, and the stocking rate. Gradient methods included the Downhill Simplex method and the David-Fletcher-Powell method. The GAs followed the general guidelines of a Simple GA by Goldberg (1989), which included the use of binary coding. Both large and small population GAs were trialled. Hart et al. commented that for low dimensionality (six decision variables, GA string length = 30), the David-Fletcher-Powell method matched the solution quality of the Simple GA and hybrid, but was considerably faster. When the problem dimensionality was increased to 15 decision variables (GA string length = 90), the gradient methods gave sub-optimal results due to an inability to escape local optima in an increasing multi-modal solution space. Hart et al. commented that the Simple GA with a small population (10) gave inferior results to a large population (50). In all cases, the use of the hybrid resulted in similar solution quality but with significantly reduced computational time. In the final stages of the Simple GA, the authors found that up to 80% of the evaluation points had been previously evaluated.

6.2.3 Conclusions

A successful approach to the problems of optimal rotational grazing and optimal farm management (that is also applicable to optimal irrigation scheduling) is the use of simulation (in time) within the objective function, decision variables describing general management principles rather than individual events, and optimisation using a heuristic method. This approach, applied to optimal irrigation scheduling, allows the modelling requirements for predicting farm profit from Chapter 2 to be met. A principal difference between the problem of optimising several different farm management decisions and optimal irrigation scheduling, is that the former will generally have greater independence between decision variables. In contrast, optimal irrigation scheduling (as the problem is structured in Sections 6.3 to 6.5) is highly constrained, with a high dependence between decision variables. Therefore, optimal irrigation scheduling is less likely to be well suited to GAs (see discussion in Section 5.4).

The main limitations of solution for these related optimisation problems are: (a) the use of SA, which optimises in continuous space, is not explored; (b) consideration is not given to more efficient methods (in place of penalty functions) for incorporating constraints; and (c) variability due to stochastic driving variables is not considered. Because of the difference and limitations, an alternative custom heuristic method, which optimises in continuous space and efficiently incorporates constraints, is developed in Chapter 8 for use in the SA scheduler. The design of the heuristic is closely coupled with the design of the scheduler structure presented in the remainder of this chapter.

6.3 SA Scheduler Overview

A novel solution for optimal irrigation scheduling, which meets the modelling and constraint requirements set forth in Chapters 1 to 3, is presented. This novel scheduler (the SA scheduler) draws on the experience of previous authors' optimal irrigation scheduling methods (Chapter 4), while considering alternative optimisation techniques (Chapter 5), to overcome weaknesses of previous schedulers. Experience is also drawn from two related optimisation problems: rotational grazing and optimum farm management (Section 6.2). The SA scheduler provides decision support during the irrigation season as to which Paddocks should be irrigated on a given day. The scheduler is run each day of the irrigation season, taking into account updated climate information. The objective is to optimise farm profit subject to constraints on the irrigation system and water supply. The key features of the SA scheduler are:

- a) Use of a farm simulation in the objective function to model farm profit;
- b) Incorporation of climate stochastic characteristics in the objective function through the simulation of multiple irrigation seasons;
- c) Optimisation of general management decisions rather than individual events;
- d) Use of a heuristic method for optimisation;
- e) Integral coupling between the design of the structure of the optimisation problem, and the design of a custom heuristic method (particularly with respect to constraints);
- f) Use of equations for evaluating mean future irrigation water use, used for meeting constraints, and adjusting irrigation management within the objective function simulation;
- g) Optimisation in continuous rather than discrete space, taking full advantage of gradient information.

In Chapter 2, the farm profit modelling requirements were discussed. From Chapter 2, it may be concluded that the complexity of the farm system likely removes the ability to retain the mathematical tractability within the objective function that is required for formal optimisation methods. Instead, the objective function must be treated as a 'black box', with irrigation strategies as the input and average annual farm profit as the output. A major component of this 'black box' is a farm system simulation, run over several irrigation seasons to account for the effects of climate variability. A similar conclusion (about the necessity of using simulation for analysing farm systems) was made by Moore

et al. (2005), who commented that “*agronomic systems are medium-number systems. They contain too many entities to be treated as small-number systems that can be solved by differential-equation techniques; and they have too few entities to be treated as large-number systems that are amenable to treatment as statistical assemblages*”. Moore et al. concluded that the best way to analyse such systems were to use simulation, which takes “*advantage of the organisation in these systems that arise from differences in the rates of different processes*”. Details of how the SA scheduler incorporates a farm simulation within the objective function is provided in Section 6.5.

Decision variables can describe either individual irrigation events or general irrigation management strategies. The problem with specifying individual events is that it greatly increases the dimensionality of the optimisation problem. Previous optimal schedulers discussed in Chapter 3 generally used decision variables that described individual events. However, the disadvantage of this approach is that either it requires over-simplifying the objective function (as in the case of dynamic programming algorithms in Sections 4.2.2 and 4.3.1), or ineffective optimisation due to the high number of decision variables (as in the case of Wardlaw and Bhaktikul’s (2004) 0/1 approach in Section 4.3.2). The alternative is that decision variables describe general irrigation strategies. These irrigation strategies should specify how to prioritise irrigation in time and space. A number of other authors, who applied heuristic methods to dairy farm simulations, have made the decision variables describe a few general management strategies rather than many individual decisions (Section 6.2). Details of how decision variables describe irrigation strategies are given in Section 6.4.

Possible methods for optimisation excludes Linear, Quadratic, and dynamic programming, due to the objective function containing a time-variant simulation. Also since individual irrigation decisions are not optimised, branch and bound methods are not suitable. Therefore, from the methods considered in Chapter 5, only gradient, bounding, and heuristic methods are suitable. Gradient and bounding methods could be used provided the number of decision variables is small. With an increasing number of decision variables, problems become highly multi-modal with many local maxima/minima (making gradient and bounding methods unsuitable), leaving heuristic methods as the only option. While heuristic methods are unable to guarantee optimality, these methods can always give solutions that equal or are better than best management practice¹³ (provided the objective function is appropriately modelled). This is because best management practice solutions can be included in the range of irrigation strategies that are explored by the heuristic method. Details of the selection and design of a custom heuristic method used within the SA scheduler is given in Chapter 8.

A flowchart of the SA scheduler is presented in Figure 6-1. Details of specific components are presented in the following sections. The SA scheduler shown is for optimal irrigation scheduling for a farm Portion (Table 2-1) – not necessarily for a whole farm (i.e. when a farm Portion is not

¹³ Best management practice refers to the current ‘state of the art’ irrigation management in New Zealand, which would typically occur when an extension service is contracted to schedule irrigation.

synonymous with a farm). In subsequent discussions, it is assumed that a farm Portion is synonymous with a farm. However, as explained in Section 2.2.4, a decompositional approach can be used (when a farm Portion is not synonymous with a farm) to scale up from several farm Portions to optimally schedule irrigation at a farm scale.

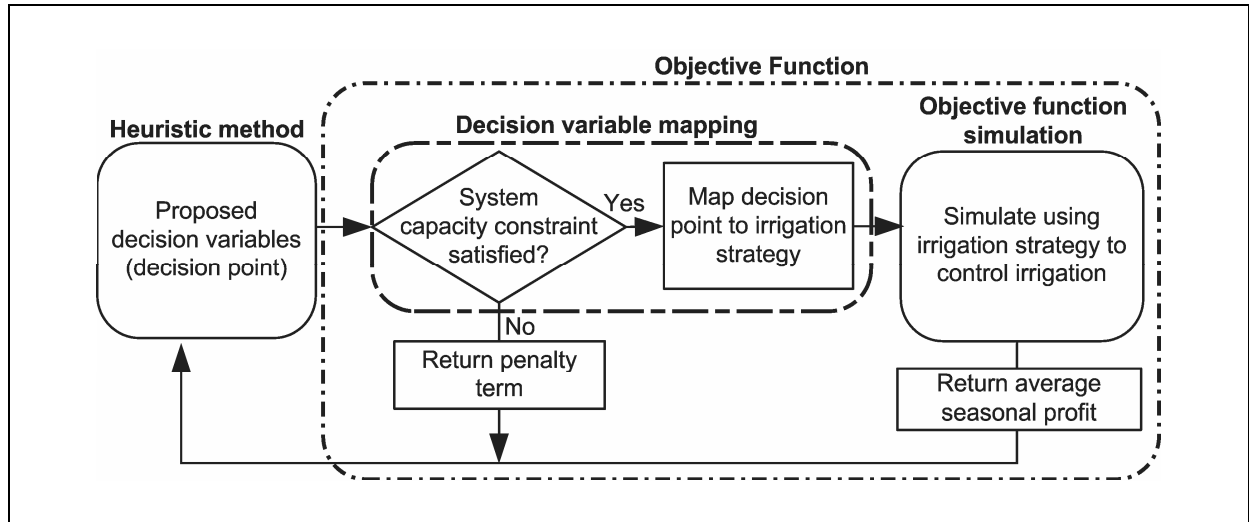


Figure 6-1: SA scheduler flowchart.

Three particular features of each set of decision variables (i.e. decision point) proposed by the heuristic method in Figure 6-1 are: (a) they are continuous; (b) they always satisfy the seasonal water use constraint; and (c) decision points that do not satisfy system capacity constraints are immediately recognised without any significant calculations. The importance of continuous variables is that this allows optimisation in continuous (rather than discrete) space, allowing for full utilisation of gradient information. Optimising in continuous space is more efficient than optimising in discrete space (Chapter 8). Efficient mechanisms for managing constraints are achieved through the coupling between the design of the structure of the optimisation problem and the design of a custom heuristic method.

Mapping of a decision point to an irrigation strategy, and adjustment of irrigation strategies within the objective function farm simulation, both require the regular use of an equation that describes the relationship between an irrigation strategy and mean future irrigation water use. This relationship is termed the *Water Use Equation*, and is explored in Chapter 7.

6.4 Decision Variable Mapping

6.4.1 Overview

Irrigation strategies describe how to prioritise irrigation in time and space. The key features of the mapping between decision variables and irrigation strategies are:

- a) Prioritisation in time is between Time Aggregation Periods (TAP) (Section 6.4.2);
- b) Prioritisation in space is between Blocks (Section 6.4.2);
- c) The priority in time and space is expressed through defining a trigger soil moisture level (at which point irrigation should occur) as a function of time, for each Block (Section 6.4.3);
- d) Decision variables are expressed in terms of water allocated to each TAP-Block permutation (Section 6.4.4);
- e) The relationship between trigger soil moisture levels and water allocated is via the Water Use Equation (Section 6.4.5);
- f) Water allocated to particular TAP and Block is adjusted proportionally (within the objective function farm simulation) to the difference between mean and simulated water use (Section 6.4.5).

Furthermore, the mapping method applies equally well to situations when the optimisation problem may be decomposed (Section 6.4.6).

6.4.2 Aggregation in time and space

Prioritisation in time is between Time Aggregation Periods (TAP). TAP would typically have a duration (length) of two weeks to two months, depending on the desired level of irrigation flexibility. While a short TAP increases the range of possible irrigation strategies explored, it will result in an increased number of decision variables, and therefore an increase in the dimensionality of the optimisation problem. Unless stated otherwise in subsequent thesis discussion, all TAPs for a given instance of the SA scheduler are of equal lengths. However, TAPs of different lengths are possible without any changes to the general performance and behaviour of the scheduler.

Within a Block, since soil and crop characteristics are the same (Table 2-2), it is reasonable to assume that when irrigator(s) are allocated to the Block on a given day; the best decision will be to irrigate the Paddocks with the greatest soil moisture deficits. Aggregating Paddocks into Blocks and prioritising between Blocks, rather than between Paddocks, greatly reduces the number of spatial units; and therefore the dimensionality of the optimisation problem.

6.4.3 Trigger soil moisture levels

Irrigation priority is expressed through defining a trigger soil moisture level as a function of time for each Block (termed a *Block Irrigation Strategy*) (Figure 6-2). The collection of Block Irrigation Strategies for the whole farm portion is termed a *Farm Irrigation Strategy*. The trigger soil moisture level (TSML) for a given TAP and Block (or Paddock) is the soil moisture trigger point at which irrigation will occur, provided an irrigator is available. Occasionally, there will be days when the soil

moisture in more than one Paddock is below the TSML but there are insufficient irrigators to immediately meet the demand. Under these circumstances, the Paddocks that have the greatest difference between their current soil moisture status and their TSML are irrigated first.

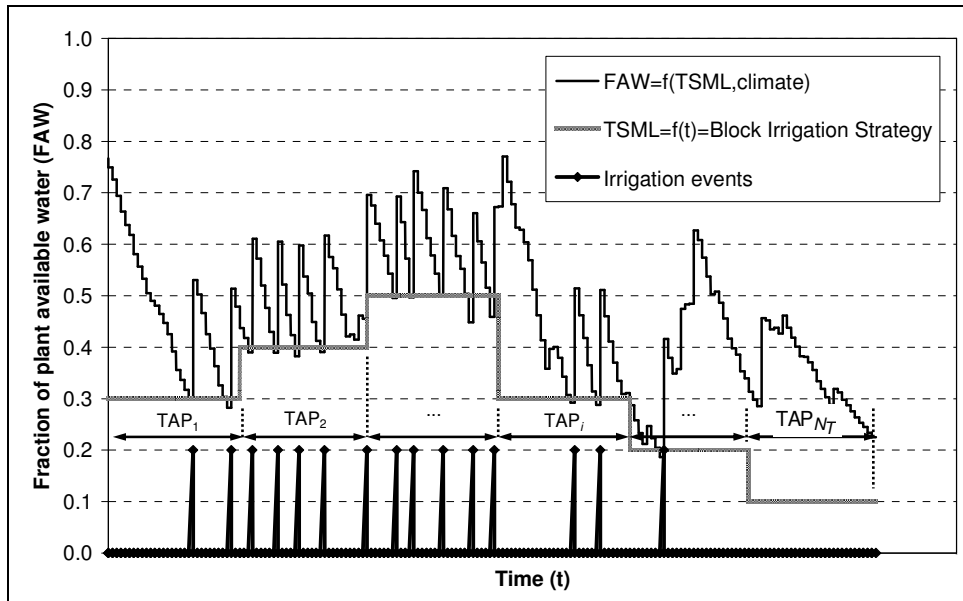


Figure 6-2: Irrigation scheduling according to trigger soil moisture levels (TSML), for a single Block, with TSML varying stepwise in time and a total of N_T steps during the irrigation season.

Under conditions of insufficient water to meet crop demands a high TSML indicates that a high priority has been given to a particular crop at a particular time of year, while a low TSML indicates a lower priority. There are three principal advantages of the TSML approach:

- Decision variables directly define a target soil moisture regime. It is the soil moisture regime, and not individual irrigation events, that affect crop development. This therefore results in a smoother objective function than using decision variables that define irrigation events directly.
- Irrigation events for an entire irrigation season can be defined with only a few decision variables (where the number of decision variables required will be dictated by the desired level of flexibility of the TSML function with respect to time).
- Since TSML describes continuous and not discrete values, there is the ability for decision variables to be continuous real valued.

TSML as a function of time is a stepwise function and can be expressed via a vector. Similarly, Farm Irrigation Strategy can be expressed via a matrix (Table 6-1).

Table 6-1: Farm trigger soil moisture levels (TSML) in matrix form.

$T_{\text{sml}}[i, j]$	Trigger soil moisture level for TAP i and Block j (proportion of TAW)
T_{sml}	Farm Irrigation Strategy
$T_{\text{sml}}[\text{column } j]$	Block j Irrigation Strategy

6.4.4 Decision variables expressed as water allocation

How seasonal water use and system capacity constraints are dealt with has a significant impact on optimisation performance. Decision variables were expressed in terms of water allocated to each TAP-Block permutation (where allocated water corresponds to the expected mean irrigation water use for a given TAP-Block). This approach allowed the seasonal water use constraint to always be satisfied (Chapter 8), and allowed strategies which do not satisfy system capacity constraints to be identified without any significant computation demand. Decision variables and associated constraints are defined in Equation 6-1 and illustrated in Figure 6-3. Decision variables are all unitless. For the illustrated example in Figure 6-3, TAPs are equal to one calendar month.

Decision variables
(decision point) = $\{\tau[N_T], \mathbf{B}[N_T, N_B]\}$

Where:

$$\tau[i] = \frac{a_\tau[i]}{SC_F} \quad [a]$$

$$\mathbf{B}[i,j] = \frac{\mathbf{A}_B[i, j]}{a_\tau[i]} \times \frac{SC_F}{SC_B} \quad [b]$$

N_T = Number of TAPs per season

N_B = Number of blocks

SWU = Seasonal water use limit (m^3)

SC_F = Farm irrigation system capacity (m^3 /TAP)

SC_B = Block irrigation system capacity (m^3 /TAP)

$$a_\tau[i] = \sum_{j=1}^{N_B} \mathbf{A}_B[i, j] = \text{Total water allocated to TAP } i \text{ (} m^3 \text{)} \quad [c]$$

$\mathbf{A}_B[i, j]$ = Water allocated to TAP i & Block j (m^3)

Subject to constraints:

$$0 \leq \tau[i] \leq 1.0 \quad [d]$$

$$0 \leq \mathbf{B}[i,j] \leq 1.0 \quad [e]$$

$$\sum_{i=1}^{N_T} \tau[i] = \frac{SWU}{SC_F} \quad [f]$$

$$\sum_{j=1}^{N_B} \mathbf{B}[i, j] = \frac{SC_F}{SC_B} \text{ for } i \in \{1, 2, \dots, N_T\} \quad [g]$$

Equation 6-1: Decision variables (expressing water allocation), and daily and seasonal water use constraints, for the proposed heuristic irrigation scheduler.

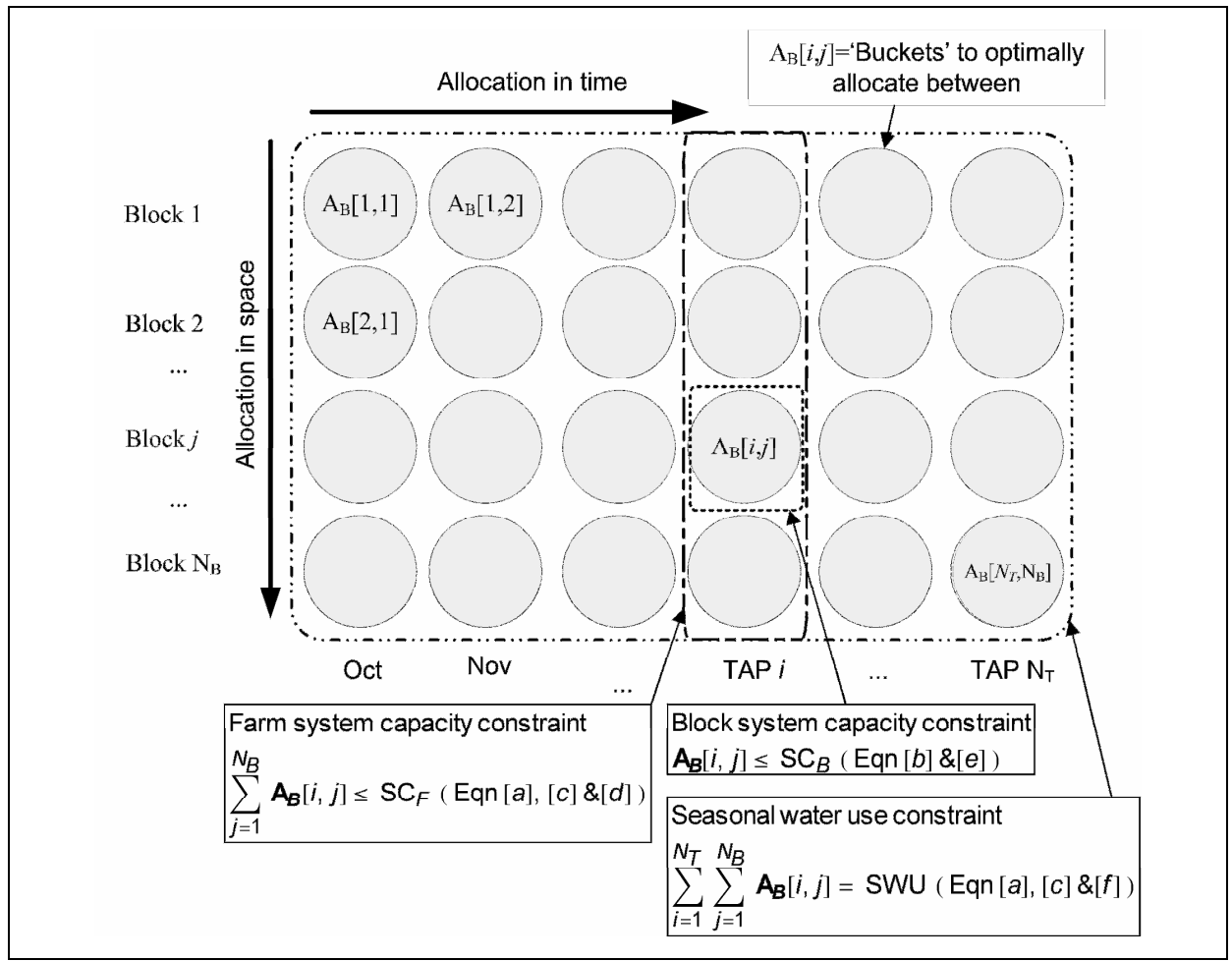


Figure 6-3: Equation 6-1 decision variables (expressing water allocation), and daily and seasonal water use constraints, for SA scheduler.

The optimisation problem becomes how best to allocate the seasonal water allocation to each of the Block-time 'buckets' (that is water allocated to a particular Block and TAP) as illustrated in Figure 6-3.

A decompositional approach was used for the seasonal water use constraint. Equation 6-1[f] specifies that the total volume of water used during the season shall be equal to the seasonal limit. However, in reality the water used during the season should be *less than* or equal to the seasonal limit. Therefore for situations where it may not be economical to use all available water, a decompositional approach is required, where the algorithm in Figure 6-1 is run for several seasonal limits. The profit versus seasonal limit relationship is then used to determine the optimal seasonal limit value. The seasonal constraint was decomposed because this approach is likely to be more computationally efficient than directly including the seasonal limit as an inequality constraint.

The reason for defining decision variables slightly differently than simply allocation divided by system capacity ($A_B[i,j]/SF_B$) relates to the design of the custom heuristic method (Chapter 8). This heuristic method not only ensures that all proposed decision points always meet equality constraints

(Equation 6-1[f] and [g]), but also uses each equality constraint to reduce the number of effective decision variables by 1 (thereby reducing problem dimensionality). The number of effective decision variables is $N_T \times N_B - 1$. Defining decision variables as $\mathbf{A}_B[i,j]/\text{SF}_B$ would result in the same number of effective decision variables (although the number of actual decision variables would be lower). However, the advantage of the definitions given in Equation 6-1 is that it results in a smoother solution space, through having some variables that describe allocation in time ($\tau[i]$) (independently of how water is allocated between blocks) and other variables that describe allocation in space ($\mathbf{B}[i,j]$). Irrigation demand (assuming efficient management) in time has a marked seasonal cycle due to the seasonal climate cycle. This smooth seasonal trend means that for most good Farm Irrigation Strategies, $\tau[i]$ will be within narrow ranges (particularly for low capacity systems). The heuristic method is able to identify and utilise this narrow range of values to increase optimisation efficiency (Chapter 8).

Other key features of the decision variable definition in Equation 6-1 are: (a) they are continuous (allowing optimisation in continuous solution space), (b) they always satisfy the seasonal water use constraint (since equality constraints are always satisfied – Chapter 8), and (c) decision points that do not satisfy system capacity constraints can be immediately recognised without any significant calculations (Equation 6-1[d] and [e] require only a simple Boolean check).

6.4.5 Relationship between allocation and trigger soil moisture levels

The relationship between trigger soil moisture levels (Table 6-1, Figure 6-2) and water allocated to a given Block and TAP (Equation 6-1) is via the Water Use Equation (Equation 6-2). Similarly, an inverse equation relates allocation to the corresponding trigger soil moisture levels.

$\mathbf{A}_B = f_{\text{WUE}}(\mathbf{T}_{\text{sml}}, \text{soil, plant and climate parameters})$	[a]
$\mathbf{T}_{\text{sml}} = f_{\text{WUE}}^{-1}(\mathbf{A}_B, \text{soil, plant and climate parameters})$	[b]
<p>Where:</p> <p style="margin-left: 40px;">f_{WUE} = Water Use Equation (Chapter 7)</p> <p style="margin-left: 40px;">f_{WUE}^{-1} = Inverse Water Use Equation (Chapter 7)</p>	

Equation 6-2: Relationship between allocation and trigger soil moisture levels.

The relationship between trigger soil moisture levels and actual irrigation water use is stochastic (varying from year to year), mainly due to the high rainfall variability (Chapter 3). However, the Water Use Equation (and its inverse) describes only the relation between trigger soil moisture levels and mean water use. This equation is used as a first approximation within simulation, for predicting future water use. When a given Farm Irrigation Strategy (expressed as \mathbf{T}_{sml} , Table 6-1) is used within

the objective function simulation, T_{sml} is adjusted periodically, in responses to differences between the actual and expected water use (Figure 6-4).

6.4.6 Decomposition

The mathematical problem of optimally allocating water (between the ‘buckets’ in Figure 6-3) cannot be decomposed with respect to time, since the marginal benefit of a given irrigation event is not independent of other irrigation events within the season. However, a spatial decomposition approach may be used when farm irrigation system capacity is high (and therefore unconstraining). In this situation, the only dependency between Blocks is the total amount of water use over the season. Therefore, water can be optimally allocated between TAPs for a given Block (independently of other Blocks) by allowing seasonal water use to be a variable. This process is repeated for each Block, resulting in a set of relationships describing Block j seasonal water use and Block j profit. These relationships can then be used to optimally allocate seasonal water between Blocks to maximise farm profit. Spatial decomposition decreases the dimensionality of the optimisation problem.

When farm system capacity is low, it becomes a constraint on irrigation scheduling during the season, and spatial decomposition cannot be used. Therefore, irrigation decisions for all Blocks throughout the season must be considered simultaneously. In this situation near constant water use over a number of months of the year is expected. A key decision is how the farm system capacity (water usage) is allocated between different Blocks. For example, the SA scheduler may recommend fully irrigating a portion of the farm during the summer, only irrigating the remaining sections of the farm during the shoulders of the season. Alternatively under-irrigating the entire farm may be advantageous. On a mixed cropping farm, the SA scheduler may be able to design a cropping pattern so that periods of high water required for one crop correspond to periods of lower water requirements for other crops.

Subsequent thesis discussion assumes decomposition between Blocks cannot be used. However, modifying the SA scheduler to allow for decomposition when farm system capacity is high, is a minor implementation issue.

6.5 Objective Function

The most important component of any optimisation procedure is the objective function. No matter how good or effective an optimisation algorithm is, if the objective function does not accurately model the real world system, confidence in the recommended management strategies will be low. Figure 6-4 further details the objective function component previously shown in Figure 6-1. The objective function in this figure has the capacity to meet the modelling and constraint requirements set forth in Chapters 1 to 3. In particular, the farm simulation can incorporate the complex model necessary for predicting farm profit (e.g. FarmWi\$e, Chapter 2). Climate stochastic characteristics (Chapter 3) are accounted for by simulating multiple (5-10) seasons of historic data.

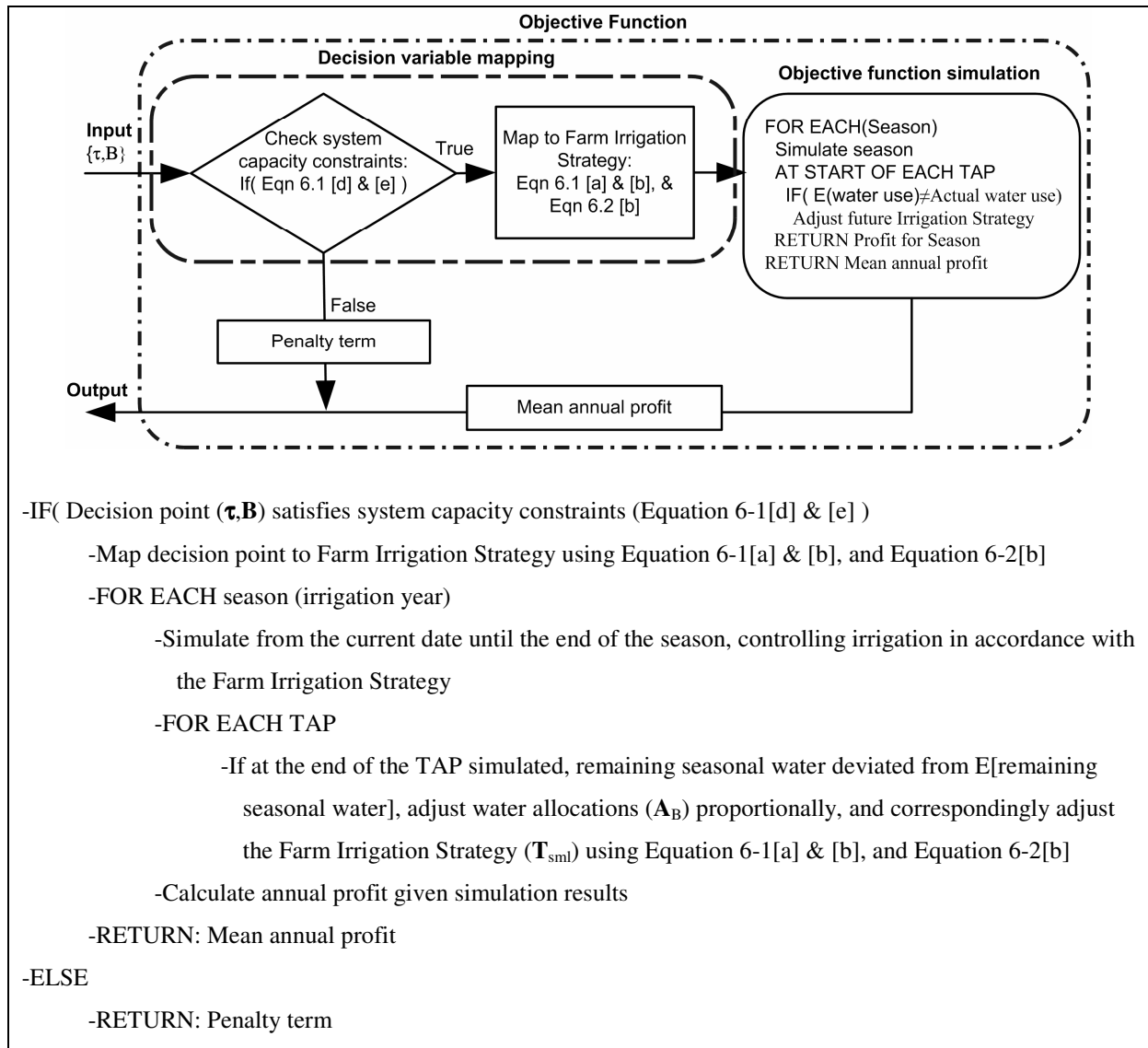


Figure 6-4: SA scheduler objective function.

Evaluating irrigation strategies via simulation is much more computationally expensive than other components of the SA scheduler; consequently, the number of farm simulation seasons will govern computational times. Therefore, the approach taken was to ensure that all strategies simulated satisfied constraints. It was not possible to avoid using a penalty term to enforce the system capacity constraint; however, non-compliant strategies were identified prior to simulation (Figure 6-4). All decision points proposed by the heuristic method automatically satisfy the seasonal water use constraint. How this is achieved is explained in Chapter 8.

Throughout the course of an objective function simulation, water allocated to a particular TAP and Block is adjusted proportionally to the difference between the expected and actual seasonal water remaining. The assumption is that this is the optimal method of adjustment. However this may not be the most optimal method of adjustment for every Farm Irrigation Strategy. The complication with trying to ensure optimality is caused by a recursive problem. To assess the performance of a particular

Strategy (given current water availability and soil moisture conditions), this Strategy needs to be trialed in simulation. However, as the simulation for a particular season progresses, the actual water use begins to deviate from the expected water use. If no adjustment to the Strategy is made, the result is that as the end of the season approaches, either the available water for the season is used up prematurely (potentially resulting in crop failure), or there is an excess of water (an opportunity loss). In order for optimal adjustments to the farm irrigation strategy (within each objective function simulation) as the expected and actual water use deviate, the SA scheduler needs to be run (inside the particular simulation for the SA scheduler that is already running) to find the new optimum management strategy for the remainder of the season. The problem is recursive; with further schedulers needing to be started inside existing schedulers, which themselves are already nested inside another scheduler. The resulting recursive problem cannot be solved. Therefore, the approach taken was to proportionally adjust the water allocation (A_B) in response to the difference between expected and actual remaining seasonal water use. To reduce the computational demand, adjustments to the Irrigation Strategy are made at the start of each TAP rather than daily.

6.6 Comparison with Previous Optimal Irrigation Schedulers

Advantages of the SA scheduler, compared with previous DP optimal scheduling algorithms (Chapter 5) are:

- a) More complex farm system models can be incorporated in the objective function allowing more accurate modelling of how farm profit is affected by irrigation management;
- b) Increasing the number of Paddocks does not increase computational time exponentially (as it does with standard DP methods);
- c) Spatial units can be aggregated into Blocks (rather than Paddocks as required by DP methods), reducing the dimensionality of the optimisation problem.

Advantages of previous DP solution are:

- a) Optimality can be guaranteed in a formal mathematical sense;
- b) In its current form, the SA scheduler does not incorporate any stochastic climate forecasting, as do some DP methods.

Advantages of the SA scheduler compared to Wardlaw and Bhaktikul's (2004) optimal scheduling algorithm (Chapter 5) are:

- a) The heuristic method is more efficient, through utilising gradient information;
- b) Constraints are dealt with in a more efficient manner;
- c) Climate is modelled stochastically (Wardlaw and Bhaktikul assumed no future rainfall);
- d) Target soil moisture levels are optimised (Wardlaw and Bhaktikul did not directly optimise any target soil moisture levels);
- e) A seasonal water use limit is allowed for (which was not allowed for by Wardlaw and Bhaktikul).

In summary, the SA scheduler is able to overcome the principle limitations of previous optimal schedulers.

6.7 Conclusions

A novel method for optimal irrigation scheduling (termed the SA scheduler) was developed. It allows for the objective function to predict farm profit using a complex farm model simulated over several years of climate data and allows for constraints on daily and seasonal water use. This scheduler was able to overcome the principle limitations of previous schedulers, in particular the inability of SDP schedulers to include complex farm models in the objective function, and the inefficient optimisation and neglect of rainfall and a seasonal water use limit by Wardlaw and Bhaktikul's (2004) heuristic scheduler. The main disadvantage of the SA scheduler is that formal optimality can not be guaranteed.

7 WATER USE EQUATIONS

7.1 Context and Overview

Chapter 6 identified the need for an equation relating Farm Irrigation Strategies (expressed in terms of trigger soil moisture levels – T_{sml}) and mean Block water use (A_B) (Equation 6.2). Two places were identified where this equation (the Water Use Equation) is used in the SA scheduler: (a) when mapping between decision variables and irrigation strategies, and (b) within the objective function simulation (to adjust Farm Irrigation Strategies in response to deviations between expected and simulated water use). This chapter derives three possible methods for the Water Use Equation: (1) simulation of multiple seasons (Section 7.3); (2) Derived probability distributions (Section 7.4); and (3) Markov chains (Section 7.5). The motivation for developing the latter two methods is computational efficiency. Efficiency of the Water Use Equation is important due to the large number of times this equation is calculated. For example, in the studies presented in Chapter 9, the Water Use Equation was used up to 10 million times per case study season simulated. Most discussion in this chapter assumes mean water use is for a single Paddock, between day a and b of the irrigation season. Up-scaling to water use for a Block (a collection of Paddocks) is discussed in Section 7.6. Most discussion is focused on mean water use as a function of trigger soil moisture levels. The Inverse Water Use Equation (where trigger soil moisture levels are a function of water use) is discussed in Section 7.7. Several areas of further work are identified throughout the chapter. The Markov chain method was shown to have significantly lower computation demands than estimating mean water use via simulation, and had several advantages over the Derived Probability Distribution method. The Markov chain was used within the SA scheduler for the irrigation scheduling case studies in Chapters 9 and 10.

7.2 Irrigation Water Use Modelling Requirements

Farm system modelling and climate modelling requirements for estimating water use were given in Chapters 2 and 3 respectively. Conclusions from these chapters are reiterated below. Three irrigation water use model variants are defined in Table 7-1, ranging from a simple model, which is acceptable under certain circumstances, to an ideal (but more complex) model, which is suitable under most circumstances.

Table 7-1: Irrigation water use model variants, ranging from a simple model (Type 1) which is acceptable for limited situations, to a more complex model (Type 3) suitable for most situations.

Model component	Irrigation water use model variant			
	Type 1	Type 2	Type 3	Additional variants
Reference ET	Mean values (as a function of the time of year)			Historic timeseries
Precipitation	Compound-Poisson			
Actual ET as a function of reference ET and soil moisture	FAO 56 single crop coefficient		FAO 56 dual crop coefficient, (or equivalent single crop coefficient with evaporation adjustments)	
Soil moisture storage	Single layer	Dual layer		
Application efficiency	Bright’s application efficiency model (Equation 2-1).			
Spatial soil moisture variability	Uniform soil moisture			Spatially variable soil moisture in response to spatial application variability
Irrigation constraints	System capacity constraints			

As discussed in Section 2.3, the FAO 56 model by Allen et al. (1998) is an appropriate and widely used method for modelling the relationship between reference and actual ET. The single crop coefficient method is adequate when the fraction of the soil surface covered by vegetation is close to 100% (closed canopy), since in this situation the single and dual crop coefficient methods are almost identical (Allen et al., Equation 76).

When the plant canopy is not fully closed, the amount of soil evaporation is affected by the frequency of rain and irrigation. The dual crop coefficient method directly accounts for this effect by modelling evaporation and transpiration separately. The single crop coefficient method could also account for this effect (to some degree) by adjusting the crop coefficient in response to irrigation and rainfall frequency. The possible advantage with the latter is an alternative method for modelling the effect of rain and irrigation frequency on evaporation in the Markov Chain Water Use Equation method.

Soil moisture storage is generally modelled as either a single or dual soil layer model. A dual layer soil model is preferable (however, single layer models are often used in practice). When water stress does not occur, irrigation water use estimates for the single and dual soil-layer models are identical. However, under deficit irrigation, the estimated water use from the two models can vary by up to 15% (Section 2.3).

Spatial non-uniformity in irrigation applications will reduce application efficiency by up to 25% (Figure 2-2). Non-uniform applications will also result in significant spatial variability in the soil moisture status following an irrigation event. Under deficit irrigation, soil moisture variability results in spatial water stress variability – resulting in reduced ET (than if water stress was spatially uniform).

The degree to which reduced ET offsets reduced application efficiency (for non-uniform irrigators) is an area requiring further research. It is desirable that both these effects be incorporated into irrigation water use modelling. An application efficiency model can easily be incorporated into all the Water Use Equation methods. However, the reduction in ET is particularly difficult to incorporate into simulation modelling (because soil moisture is required to be a probabilistic variable) and the Derived Probability Distribution method, but can be easily incorporated into the Markov chain method (Section 7.5.10).

Approximating a historic rainfall timeseries with a compound-Poisson (with Gamma depth distributions) and approximating a historic reference ET timeseries with mean values (which are a function of time of year) have minimal impact on cumulative mean irrigation water use. Together, both of these approximations result in about a 3% increase in predicted water use (Section 3.6).

The influence of incorporating additional model variants (historic timeseries and/or spatial soil variability) is expected to be small; however, further work is required to quantify the effect of spatial soil variability. The negative impact on the SA scheduler's performance from incorporating simpler (but less accurate) models within the Water Use Equation (e.g. Type 1) has not been assessed.

7.3 Simulation of multiple seasons

The standard method for predicting irrigation water use is via a farm simulation. This method can easily incorporate Type 1 to Type 3 irrigation water use models (Section 7.2), with minimal differences in the computational demand. The ability to incorporate historic climate is dependent on the number of years of available data and the required level of accuracy, which is determined by the number of seasons simulated. It is not possible to include spatial soil variability without a large increase in computational demand. The two disadvantages of using simulation for Type 1 to Type 3 irrigation water use models are (a) the computational demand is high, and (b) the relationship between the TSML and mean water use is not smooth.

The high computational demand is due to the stochastic climate input, which requires that multiple seasons be simulated. The discontinuous relationship between TSML and mean water use is a result of stochastic climate and discrete irrigation application depths. Discontinuities increase when irrigation application depths are large and/or when a small number of seasons are simulated. Discontinuities decrease the efficiency of iterative root finding procedures used in the Water Use Equation and its inverse.

Using the central limit theorem (for the estimate of a population mean from a sample), the error when estimating the long-term mean water use (population mean) decreases as the number of seasons simulated (sample size) increases. From Chapter 3 trends and cycles greater than 1 year do not significantly affect water use. Therefore, it is reasonable to assume that consecutive irrigation seasons

are approximately independent; hence the central limit theorem assumption of a stationary and random process is valid. The estimated computation demands, given a range of error tolerance levels, are presented in Table 7-2. Simulation parameters used in this analysis are as per Table 3-7. The case study below is for deficit irrigation, which is conservative, since errors (expressed as a percentage) are greater for deficit irrigation¹⁴.

Table 7-2: Computational demand for estimating mean seasonal irrigation water use from simulation.

Parameter	Value			
Mean seasonal water use ⁽¹⁾	315 mm			
Std. dev. seasonal water use ⁽¹⁾	39.5 mm			
Computation time per season ⁽²⁾	2 ms			
Allowable error ⁽³⁾	2%	5%	10%	15%
No. seasons to simulation ⁽⁴⁾	106	17	4	2
Total computational time	212 ms	34 ms	8 ms	4 ms
<p>(1) Table 3-7, simulation run ID 2.2.</p> <p>(2) Based on FarmSim (Good 2005) running on a P4, 2.8 GHz processor, and applicable for Type 1 to Type 3 irrigation water use models since the difference in computational demand between these models is minimal.</p> <p>(3) Maximum allowable error for most (~95%) of calculations.</p> <p>(4) From the Central Limit Theorem, the required number of simulation seasons (n) for a 90% confidence interval is:</p> $n = \left(\frac{1.645 \sigma}{\Delta \bar{x}} \right)^2$ <p>Where:</p> <p>\bar{x} = Mean water use</p> <p>σ = Std.dev.water use</p> <p>Δ = Allowable error %</p> <p>A 90% (in place of 95%) confidence interval is used since error estimates are already conservative as a result of simulating only deficit irrigation. The standard deviation in annual water use is greater for deficit irrigation than for non-deficit irrigation.</p>				

Errors (expressed as a percentage) at a monthly time scale are greater than at a seasonal time scale. Table 7-3 presents the water use variability at a monthly time scale (relative to a seasonal time scale). The simulation studies used to generate these figures are the same as those used to produce Table 7-2. The coefficient of variation is directly proportion to the number of seasons that need to be simulated to achieve a pre-defined allowable error. This means that in order to achieve the same allowable error of (e.g. 5%); mean monthly water use estimates require a significantly greater number of seasons be simulated than when estimating only mean seasonal water use. Computational demands are therefore proportionally higher.

¹⁴ Compare Table 3-8, ID 2.1 and ID 2.2

Table 7-3: Computational demand for estimating mean monthly water use from simulation.

Relative water use variability	Sep	Oct	Nov	Dec	Jan	Feb	Mar
CoV[Month] CoV[Season]	13.3	3.9	2.5	1.9	1.9	2.2	3.6
For an allowable error of 5% (refer Table 7-2)							
No. seasons to simulation	226	67	43	33	33	38	62
Total computational time (ms)	452	134	86	66	66	76	124

The important conclusion from Table 7-2 and Table 7-3 is that there is a significant computational demand associated with achieving a reasonable accuracy in water use estimates. The Water Use Equation requires the relationship between the TSML for a given TAP and the mean water use for that TAP. Since TAPs range from two weeks to two months (Section 6.4.2), the computational times in Table 7-3 will be indicative of the Simulation Water Use Equation computational requirements (assuming an allowable error of 5% in TAP mean water use estimates is appropriate). Further work is required to assess how allowable errors affect the SA scheduler's performance, so that an appropriate allowable error can be specified.

7.4 Derived Probability Distribution Method

7.4.1 Overview

The Derived Probability Distribution (DPD) Water Use Equation assumes a Type 1 water use model but with exclusion of Bright's application uniformity model. It is based on DPDs for soil moisture, ET and rainfall, which assumes a continuous time domain. This method assumes that the soil moisture probability density function is independent of the timing and depth of specific rain events. This assumption was tested via a simulation study. The method was not extended to include Type 2 or Type 3 irrigation water use models.

7.4.2 Seasonal water balance

Mean Paddock water use between days of the irrigation season a to b is given by the mass balance in Equation 7-1.

$$\overline{w_u} = \overline{e_{a\Sigma}} - \overline{ue_{\Sigma}} + \overline{\theta_{\Delta}}$$

Where:

$\overline{w_u}$ = Mean Paddock cumulative irrigation water use between days of the irrigation season a to b

$\overline{e_{a\Sigma}}$ = Mean cumulative ET, from a to b (mm) – Equation 7-7

$\overline{ue_{\Sigma}}$ = Mean cumulative effective rainfall, from a to b (mm) – Equation 7-8

$\overline{\theta_{\Delta}}$ = $\overline{\theta_b} - \overline{\theta_a}$ = Expected change in soil moisture between a and b

and $\overline{\theta_x}$ = Expected soil moisture on day x (mm) – Equation 7-6

Equation 7-1: Water balance for estimating the mean irrigation water use.

7.4.3 Soil moisture depletion

Equation 7-2 gives an equation for actual ET, given a Type 1 irrigation water use model.

$$e_a = k_s e_p \quad [a]$$

And

$$k_s = \begin{cases} 1 & \text{RAW} < \theta \leq \text{TAW} \\ \frac{\theta}{\text{TAW} - \text{RAW}} & 0 \leq \theta \leq \text{RAW} \end{cases} \quad [b]$$

Where

e_p = Potential evapotranspiration (mm/day) = $k_c e_{t_o}$

e_a = Actual evapotranspiration (mm/day)

k_s = Water stress reduction coefficient (unitless, 0-1.0)

k_c = ET single crop coefficient (Allen et al. 1998) (unitless)

θ = Plant available soil moisture (mm)

TAW = (Plant) Total Available Water (mm)

RAW = (Plant) Readily Available Water (mm)

Equation 7-2: Actual ET.

In the absence of rainfall or irrigation, the soil moisture reduces linearly until the soil moisture deficit is greater than the Readily Available Water. Beyond this point, soil moisture reduces exponentially, asymptotically approaching the permanent wilting point ($\theta = 0$). In semi-arid environments, the influence of rainfall is to slow the depletion process. Without irrigation, with time, the soil moisture levels will stabilise at a level where actual ET is equal to the average daily depth of rainfall.

When irrigation is used to maintain the soil moisture within some target range, the soil moisture approximately follows a ‘saw tooth’ pattern, where rapid recharge from irrigation is followed by gradual soil depletion from ET, which is slowed intermittently by rainfall events (Figure 7-1).

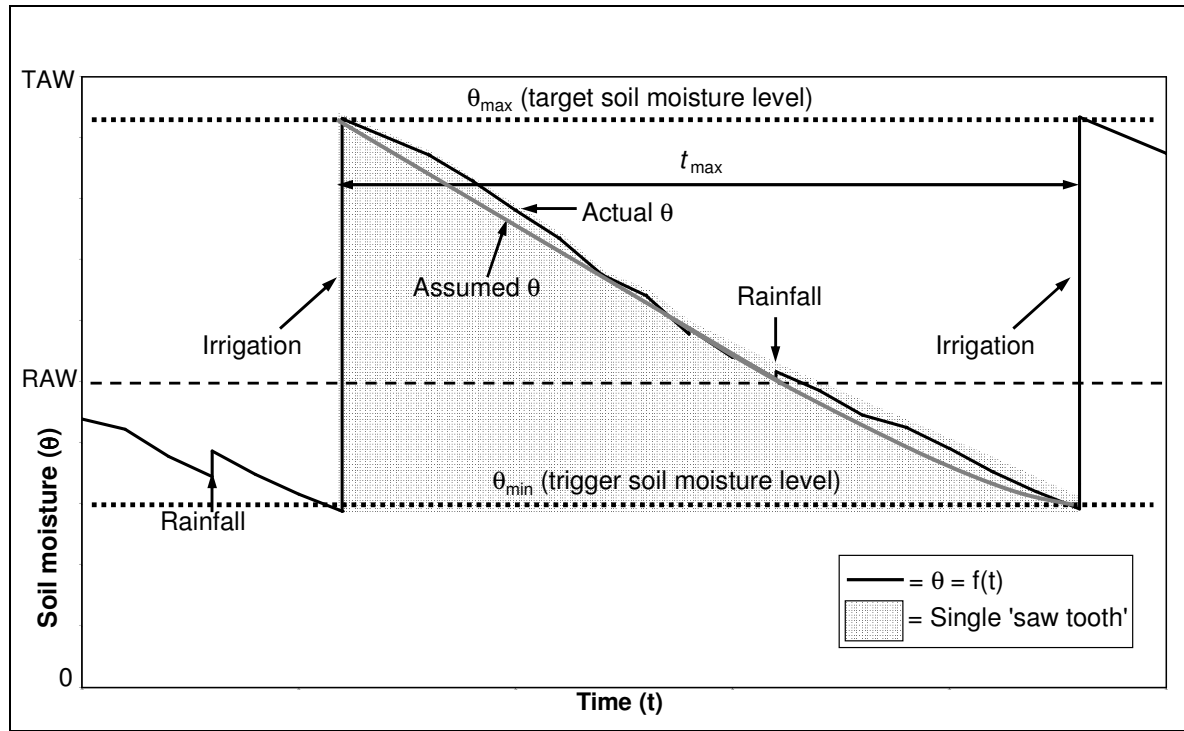


Figure 7-1: Soil moisture depletion process after an irrigation event.

In Figure 7-1, the irrigation depth is equal to the target soil moisture level minus the trigger soil moisture level. Equation 7-3 describes the soil moisture depletion process in time (for assumed θ), for one 'saw tooth' – extending from one irrigation event to the next (Figure 7-1). The assumptions made in Equation 7-3 is that rainfall depths are small relative to the irrigation application depth, and that mean reference ET is greater than the mean daily rainfall. Given these assumptions runoff and deep drainage may be ignored. These assumptions were also required so that the soil moisture probability density function (PDF) in Equation 7-4 (which is derived from Equation 7-3) is independent of the timing and depth of specific rain events. These assumptions are tested in Section 7.4.8.

Another assumption made in Equation 7-3 (and subsequent equations in Section 7.4) was that $\theta_{\max} > \text{TAW-RAW}$ and $\theta_{\min} < \text{TAW-RAW}$. While it is possible to derive Equation 7-3 and subsequent equations without the assumption, this was not done as part of this thesis, since while developing different approaches to the Water Use Equation, the Markov chain approach was found to be superior. While drainage was ignored when deriving the soil moisture PDF, it was considered when calculating the effective rainfall in Section 7.4.6.

The average rate of soil moisture depletion when rainfall events are small is:

$$\frac{d\theta}{dt} \sim -(e_p - r) \quad [a]$$

Solving $d\theta/dt$, together with Equation 7-2, with initial conditions $\theta(t=0) = \theta_{\max}$ and $\theta(t=t_{\max}) = \theta_{\min}$:

$$\theta = \begin{cases} \theta_{\max} - (e_p - r) t & 0 \leq t < \frac{\theta_{\max} - C_1}{e_p - r} \\ \frac{C_1}{e_p} \left(r + (e_p - r) \exp\left(\frac{e_p(\theta_{\max} - C_1 - t(e_p - r))}{(e_p - r) C_1}\right) \right) & \frac{\theta_{\max} - C_1}{e_p - r} \leq t \leq t_{\max} \end{cases} \quad [b]$$

And

$$t_{\max} = \frac{\theta_{\max} - C_1}{e_p - r} + \frac{C_1 C_2}{e_p} \quad [c]$$

Where:

- r = Mean daily rainfall (mm/day)
- θ_{\min} = Trigger [plant available] soil moisture level (mm) (= TSML x TAW)
- θ_{\max} = Target [plant available] soil moisture level (mm)
- t = Time (days)
- C_1 = TAW-RAW
- $C_2 = \ln\left(\frac{C_1(e_p - r)}{e_p \theta_{\min} - r C_1}\right)$

Equation 7-3: Soil moisture depletion in time following an irrigation event.

7.4.4 Soil moisture distribution

The probability that the soil moisture will be within a certain range on a randomly selected day is given by the portion of time spent in that soil moisture range. This can be derived from Equation 7-3 (which can be considered as one period of the periodic function of soil moisture) and is used in Equation 7-4 to obtain the soil moisture PDF.

Rearranging Equation 7-3 to express time as a function of expected soil moisture (t_θ):

$$t_\theta = \begin{cases} \frac{\theta_{\max}-C_1}{e_{p-r}} + \frac{C_1}{e_p} \ln\left(\frac{C_1(e_{p-r})}{e_p \theta - r C_1}\right) & \theta_{\min} \leq \theta < C_1 \\ \frac{\theta_{\max}-\theta}{e_{p-r}} & C_1 \leq \theta \leq \theta_{\max} \end{cases} \quad [\text{a}]$$

Using t_θ to obtain the soil moisture PDF. The probability the random variable X is between $(\theta-\Delta)$ and $(\theta+\Delta)$ is given by:

$$P[\theta-\Delta \leq X \leq \theta+\Delta] = \frac{t_\theta(\theta + \Delta) - t_\theta(\theta - \Delta)}{t_{\max}} \quad [\text{b}]$$

Likewise:

$$P[X \leq \theta] = 1 - P[X \geq \theta] = 1 - \frac{t_\theta(\theta)}{t_{\max}} \quad [\text{c}]$$

Therefore:

$$\text{PDF}_\theta = \frac{\frac{dt}{d\theta}}{t_{\max}} = \begin{cases} \frac{C_1}{(e_p \theta - r C_1) \left(\frac{\theta_{\max}-C_1}{e_{p-r}} + \frac{C_1 C_2}{e_p} \right)} & \theta_{\min} \leq \theta \leq C_1 \\ \frac{1}{\theta_{\max} + C_1 \left(\frac{C_2(e_{p-r})}{e_p} - 1 \right)} & C_1 \leq \theta \leq \theta_{\max} \end{cases} \quad [\text{d}]$$

Where:

PDF_θ = Soil moisture PDF (unitless)

Δ = Infinitesimal

Equation 7-4: Soil moisture PDF.

The PDF for the soil moisture deficit (θ_d) in Equation 7-5 was derived by substituting $\theta_d = \text{TAW} - \theta$ into Equation 7-4. Both the soil moisture and the soil moisture deficit PDF are illustrated in Figure 7-2. The expected soil moisture, obtained from integrating Equation 7-4, is given in Equation 7-6.

$$\text{PDF}_{\theta_d} = \begin{cases} \frac{1}{(e_{p-r}) \left(\frac{\theta_{\max}-C_1}{e_{p-r}} + \frac{C_1 C_2}{e_p} \right)} & \text{TAW} - \theta_{\max} \leq \theta_d \leq \text{RAW} \\ \frac{C_1}{(e_p(\text{TAW} - \theta_d) - C_1 r) \left(\frac{\theta_{\max}-C_1}{e_{p-r}} + \frac{C_1 C_2}{e_p} \right)} & \text{RAW} \leq \theta_d \leq \text{TAW} - \theta_{\min} \end{cases}$$

Where:

θ_d = Soil moisture deficit = $\text{TAW} - \theta$

PDF_{θ_d} = Soil moisture deficit PDF (unitless)

Equation 7-5: Soil moisture deficit PDF.

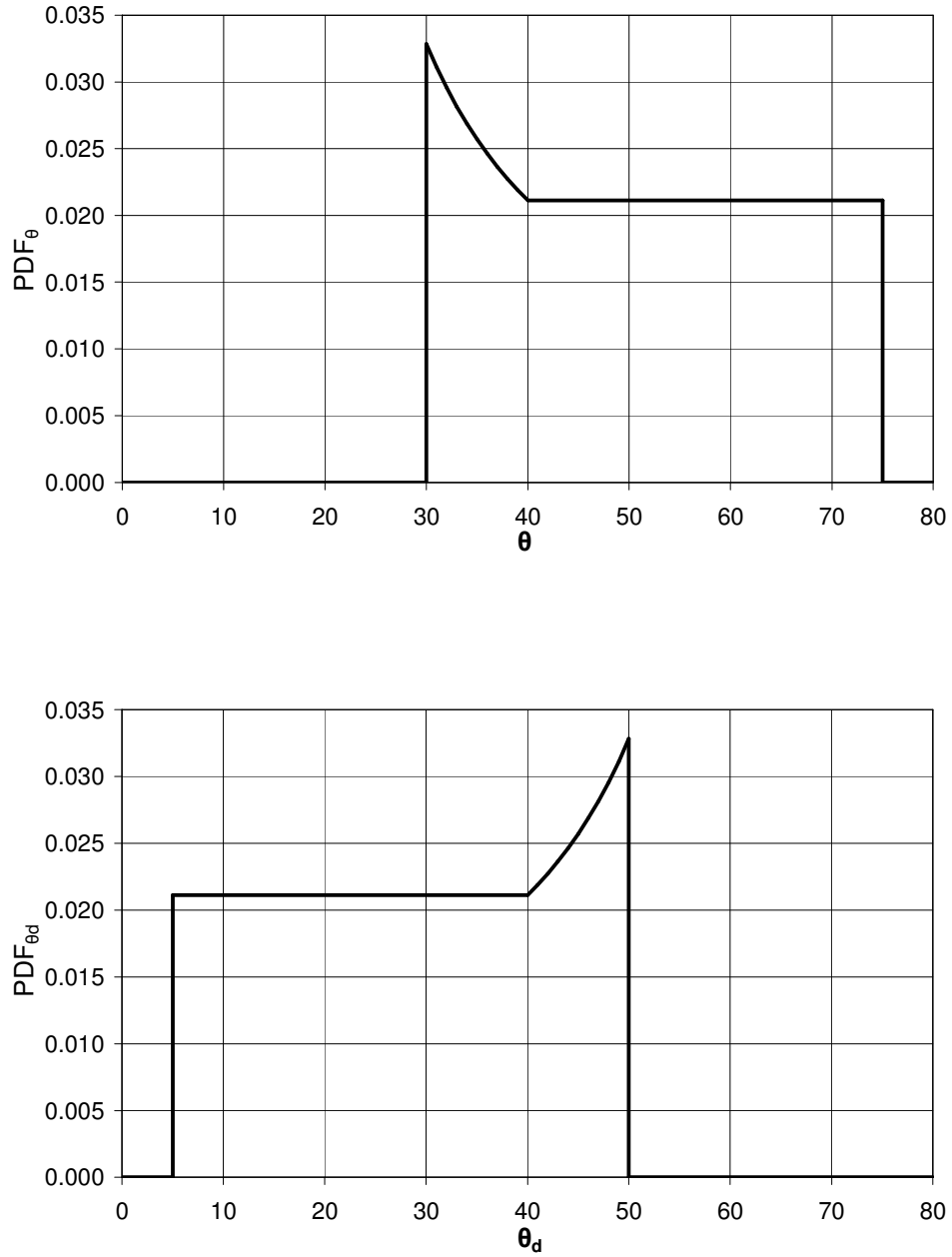


Figure 7-2: Soil moisture and soil moisture deficit PDFs, given TAW = 80 mm, RAW = 40 mm, $\theta_{max} = 75$ mm, $\theta_{min} = 30$ mm, $r = 1.5$ mm/d, $e_p = 5$ mm/d.

$$\begin{aligned}\bar{\theta} &= \int_{\theta_{min}}^{\theta_{max}} \text{PDF}_{\theta} d\theta \\ &= \frac{e_p \left(-C_1^2 + \theta_{max}^2 + \frac{1}{e_p^2} 2 C_1 (e_p - r) (C_1 (e_p + C_2 r) - e_p \theta_{min}) \right)}{2 (C_1 (-1 + C_2) e_p - C_1 C_2 r + e_p \theta_{max})}\end{aligned}$$

Where:

$\bar{\theta}$ = expected soil moisture (mm)

Equation 7-6: Expected soil moisture.

7.4.5 Mean cumulative ET

Mean total ET (from days of the irrigation season a to b) in Equation 7-7 was derived from Equation 7-2 and Equation 7-4.

$$\overline{e_{a_{\Sigma}}} = \sum_{i=a}^b \overline{k_{s_i}} e_{p_i} \quad [\text{a}]$$

And for each i day:

$$\begin{aligned} \overline{k_s} &= \int_{\theta_{\min}}^{\theta_{\max}} k_s \text{PDF}_{\theta} d\theta \\ &= \frac{e_p (e_p (\theta_{\max} - \theta_{\min}) - r (C_1 - \theta_{\min})) + r C_1 (e_p - r) (\ln((e_p - r) C_1) - \ln(e_p \theta_{\min} - r C_1))}{e_p (e_p (\theta_{\max} - C_1) - (e_p - r) C_1 C_2)} \quad [\text{b}] \end{aligned}$$

Where:

$\overline{e_{a_{\Sigma}}}$ = Mean cumulative ET between days of the irrigation season a to b (mm)

$\overline{k_s}$ = Mean water stress factor (unitless)

Equation 7-7: Mean cumulative ET.

7.4.6 Seasonal effective rainfall

Rainfall was assumed to be a compound-Poisson process, as described in Equation 3-3. Not all rainfall is effective, since drainage may occur, particularly when a rain event occurs when the soil moisture is already high. The total effective rainfall in Equation 7-8 was derived from Equation 3-3 and Equation 7-5.

Assuming uniform infiltration and soil properties (UCC = 1.0), the proportion of rainfall that is effective (q) is given by:

$$q = \begin{cases} 1 & 0 \leq \frac{u}{\theta} \leq 1 \\ \frac{\theta}{u} & 1 < \frac{u}{\theta} < \infty \end{cases} \quad [\text{a}]$$

Mean effective rainfall is given by:

$$\begin{aligned} \overline{u_e} &= \int_{\text{TAW}-\theta_{\max}}^{\text{TAW}-\theta_{\min}} \left(\int_0^{\infty} q u \text{PDF}_u d u \right) \text{PDF}_{\theta_d} d \theta_d \\ &= \int_{\text{RAW}}^{\text{TAW}-\theta_{\min}} \frac{e_p (e_p - r) C_1 (\Gamma(\beta + 1) + \alpha \theta_d \Gamma(\beta, a \theta_d) - \Gamma(\beta + 1, a \theta_d))}{\alpha (E_p (\text{TAW} - \theta_d) - r C_1) \Gamma(\beta) (E_p (\theta_{\max} - C_1) - (E_p - r) C_1 C_2)} d \theta_d + \\ &\quad (e_p ((\text{RAW}^2 \Gamma(\beta, a \text{RAW}) - (\text{TAW} - \theta_{\max})^2 \Gamma(\beta, a (\text{TAW} - \theta_{\max}))) \alpha^2 + \\ &\quad 2 ((\theta_{\max} - C_1) \Gamma(\beta + 1) - \text{RAW} \Gamma(\beta + 1, a \text{RAW}) + \\ &\quad a (\text{TAW} - \theta_{\max}) \Gamma(\beta + 1, a (\text{TAW} - \theta_{\max}))) + \\ &\quad \Gamma(\beta + 2, a \text{RAW}) - \Gamma(\beta + 2, a (\text{TAW} - \theta_{\max}))) / \\ &\quad (2 a \Gamma(\beta) (a e_p (\theta_{\max} - C_1) - \alpha (e_p - r) C_1 C_2)) \end{aligned} \quad [\text{b}]$$

Total mean cumulative rainfall (from days of the irrigation season a to b) is given by:

$$\overline{u_{e\Sigma}} = \sum_{i=a}^b \overline{u_{e_i}} \lambda_i \quad [\text{c}]$$

Where:

u = Rainfall (infiltration) depth (mm)

$\overline{u_e}$ = Mean effective rainfall depth (mm)

$\overline{u_{e\Sigma}}$ = Mean cumulative effective rainfall between days of the irrigation season a to b (mm)

PDF_u = Rainfall depth PDF (unitless) – Equation 3-3

λ = Mean number of storm events per day

α = Gamma scale parameter for storm depths (mm^{-1}) ($= \alpha_s$)

β = Gamma shape parameter for storm depths (mm^{-1}) ($= \beta_s$)

$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt$ = Euler's Gamma function

$\Gamma(a, z) = \int_z^{\infty} t^{a-1} e^{-t} dt$ = Incomplete Gamma function

Equation 7-8: Mean total effective rainfall.

Equation 7-8 requires a portion of numerical integration. The function being integrated is smooth and well behaved, and therefore the computational demand from this integration is low.

7.4.7 System capacity

System capacity can be modelled by specifying that the mean water use for a given TAP cannot exceed the system capacity (expressed as mm/TAP). When an initial TSML results in mean water use greater than the system capacity, the TSML may be iteratively adjusted until mean water use is equal to the system capacity.

7.4.8 Testing via simulation

Performance of the DPD Water Use Equation was tested by comparing DPD Water Use Equation estimates of actual ET, effective rainfall, change in soil moisture, and mean irrigation water use calculated from timeseries simulation. Simulation parameters are presented in Table 7-4, and the results are presented in Table 7-5.

Table 7-4: Simulation case study parameters for testing the performance of the DPD Water Use Equation.

Parameter	Value
Model description	Type 1 water use model (without Bright's application efficiency model)
Crop	Pasture – constant rooting depth and crop coefficient of 1.0
Soil TAW (mm)	80
Soil RAW (mm)	40
Irrigation depth (mm)	$(\theta_{\max} - \theta_{\min})$ - as per Table 7-5
Trigger soil moisture level	θ_{\min} - as per Table 7-5
System capacity	8 mm/day (unconstraining)
Case study irrigation years	500 years of synthetically generated compound-Poisson rainfall and standardised ETo timeseries

Table 7-5: Water use estimates from simulation and the DPD Water Use Equation.

ID	Target θ regime		Simulation				DPD Equation				$\overline{w_u}$ % Δ
	θ_{min}	θ_{max}	$\overline{e_{a\Sigma}}$	$\overline{u_{e\Sigma}}$	$\overline{\theta_\Delta}$	$\overline{w_u}$	$\overline{e_{a\Sigma}}$	$\overline{u_{e\Sigma}}$	$\overline{\theta_\Delta}$	$\overline{w_u}$	
<i>Irrigation season: 1 November – 28 February</i>											
1	25	80	519.6	135.9	48.0	431.7	515.4	130.3	47.1	432.2	0.1%
2	40	80	550.0	128.4	55.8	477.1	555.6	121.0	56.9	491.5	-3.0%
3	25	65	509.4	144.6	41.2	406.0	502.0	144.0	39.8	397.8	2.0%
4	20	45	477.3	148.1	32.2	361.4	461.4	147.1	30.6	344.9	4.6%
5	10	40	489.1	148.2	32.4	373.3	481.3	147.2	31.5	365.6	2.1%
<i>Irrigation season: 1 October – 29 March</i>											
6	25	80	694.4	202.7	1.5	493.2	683.3	197.7	-3.9	481.7	2.3%
7	40	80	734.6	192.3	7.9	550.2	739.2	181.8	8.0	565.4	-2.8%
8	25	65	682.6	219.1	-1.6	461.9	665.5	219.5	-10.9	435.1	5.8%
9	20	45	642.7	223.4	-11.2	408.1	611.0	225.0	-19.4	366.6	10.2%
10	10	40	662.6	223.4	-10.4	428.8	639.7	225.0	-17.6	397.1	7.4%
<u>Symbols & abbreviations:</u> θ = Soil moisture (mm) θ_{min} = Trigger soil moisture level (Figure 7-1), (mm) θ_{max} = Target soil moisture level (Figure 7-1), (mm) $\overline{e_{a\Sigma}}$ = Mean cumulative actual ET over the irrigation season (Equation 7-7), (mm) $\overline{u_{e\Sigma}}$ = Mean cumulative effect rainfall over the irrigation season (Equation 7-8), (mm) $\overline{\theta_\Delta}$ = Mean difference in soil moisture, between the start and end of the irrigation season (Equation 7-1), (mm) $\overline{w_u}$ = Mean cumulative seasonal irrigation water use (Equation 7-1), (mm) % Δ = Percent difference											

The DPD Water Use Equation derivation assumed that the soil moisture PDF was independent of the timing and depth of specific rain events. Results from Table 7-5 show that this assumption is generally reasonable. The validity of this assumption, and therefore the accuracy of the DPD equation, is poorer when the irrigation season included the shoulder months of October and March, compared with when irrigation only occurs during the peak of the season. During the shoulders of the season, the ratio of ET to mean daily rainfall is low. Performance was also poorer for more deficit irrigation regimes and/or small irrigation application depths.

7.4.9 Implementation issues

An efficient implementation of the DPD Water Use Equation would calculate Equation 7-1, Equation 7-6, Equation 7-7[b] and Equation 7-8[b] once for each TAP (where the climate, soil and irrigation regime characteristics would be assumed to be constraint within a TAP). The expected change in water use ($\overline{\theta_\Delta}$) for a given TAP is given by the expected soil moisture from the previous TAP (Equation 7-6) minus the expected soil moisture for the given TAP. Changes in rooting depth between TAPs require a corresponding change in plant available water. Computational times have not been

quantified but can be expected to be low, relative to the Simulation and Markov Chain Water Use Equations, given the solution is largely analytical.

7.4.10 Incorporating Type 2 and Type 3 irrigation water use models

Given the advantages of the Markov chain method over the DPD method, no further attempts were made to include more complex components than allowed for in the Type 1 irrigation water use model. Furthermore, extending this method to include a Type 2 irrigation water use model (defined in Section 7.2) is likely to be difficult.

7.5 Markov Chain Method

7.5.1 Overview

A Water Use Equation that uses a first order Markov chain is described below. The major advantage of this method over the DPD method is that it is not necessary to make the assumption that the soil moisture PDF is independent of the timing and depth of specific rain events. This Markov Chain Water Use Equation is an exact solution of the mean irrigation water use, given a Type 1 irrigation water use model. The conclusion is illustrated by a simulation study, where calculated mean water use via simulating a large number of seasons was identical to water use from the Markov chain equation. The Markov chain method also has potential to be extended to Type 2 and Type 3 irrigation water use models.

7.5.2 Background

The Markov chain method used to estimate mean water use is partially based on the stochastic dynamic programming formulation by Cordova and Bras (1979), who used a first-order Markov chain to describe the response of soil moisture to ET, rainfall and drainage. The authors used a single layer water balance model, and defined the soil moisture as the state variable. The recharge and depletion of soil moisture by external processes was expressed through Probability Transition Matrixes (PTMs) acting on the soil moisture Probability Vector (PV). Reference ET was modelled deterministically dependent only on the time of year, while actual ET essentially used the FAO 56 (Allen et al. 1998) single crop coefficient method. Rainfall was modelled stochastically using the same compound-Poisson process described in Chapter 3. Their soil moisture storage model differed slightly from the ‘bucket’ model presented in Section 2.1.6, with Cordova and Bras allowing for non-instantaneous drainage of soil moisture greater than field capacity.

The irrigation water use model assumed for the Markov Chain Water Use Equation is a Type 1 model (Section 7.2). This model is identical to that assumed by Cordova and Bras, with the exception that deep drainage is assumed to be instantaneous (a variant likely to make little difference given the rapid rate of drainage to field capacity in most Canterbury soils). Following the approach of Cordova and Bras, soil moisture was used as the state variable. Soil moisture was discretised such that the amount of depletion from actual ET over one time unit resulted in soil moisture decreasing by exactly one

state interval. Irrigation scheduling was modelled using trigger soil moisture levels, where irrigation occurs when soil moisture drops below a defined threshold. A summary of the PTMs for these system components are presented in the following sections. Naming conventions (for vectors and matrixes) used in this thesis are:

- Vectors = Bold, lower case; element reference $\mathbf{a}[i]$; dimensions $\mathbf{a}[x] = x$ elements
- Matrixes = Bold, upper case; element reference $\mathbf{A}[i,j]$; dimensions $\mathbf{A}[x,y] = x$ rows and y columns

7.5.3 Soil moisture distribution

The soil moisture PV at any time during the irrigation season is given in Equation 7-9.

$$\theta_{pv_b} = \theta_{pv_a} \prod_{i=a}^b \mathbf{I}_{g_i} \mathbf{E}_{a_i} \mathbf{U}_{e_i} \Theta_i$$

Where:

- a = Given day of season (for which the probabilistic soil moisture status is known)
- b = Future day of season (for which the probabilistic soil moisture status is required)
- θ_{pv_x} = Soil moisture (distribution) PV on day x (unitless)
- \mathbf{I}_g = Irrigation PTM (unitless) – Equation 7-13
- \mathbf{E}_a = Actual ET PTM (unitless) – Equation 7-14
- \mathbf{U}_e = Effective precipitation PTM (unitless) – Equation 7-14
- Θ = Transformation matrix for re-discretisation of θ_{pv} (unitless) – Equation 7-12

Equation 7-9: Soil moisture Probability Vector.

The soil moisture PV (θ_{pv}) is a discretised version of the soil moisture PDF (PDF_θ), previously used in Equation 7-4. As the time discretisation interval and truncation error (defined below) tend to zero (hence the time domain is continuous), θ_{pv} has an identical definition to PDF_θ . PTMs describe how the soil moisture distribution (θ_{pv}) changes in response to some event. The elements of the matrix $\text{PTM}[i,j]$ define the probability of the event changing the soil moisture state i to state j . A particular characteristic of PTM is that the sum of row elements always adds to 1.0.

For the purposes of predicting irrigation water use, the soil moisture distribution is required only during the irrigation season. However, outside the irrigation season, Equation 7-9 is still valid with the exception that the irrigation PTM is equal to the identity matrix (i.e. has no influence on soil moisture). An overview of each of the PTM is presented in the following sections in the order they appear in Equation 7-9. Further details of calculating these PTMs and Θ are given in the electronic copy of the source code appended on CD. Mean water use for any period of the irrigation season is given in Equation 7-10.

$$\overline{w_u} = \sum_{i=a}^b \theta_{pv_i} \cdot i_{g_i}$$

Where:

$\overline{w_u}$ = Mean Paddock water use between days of the irrigation season a to b

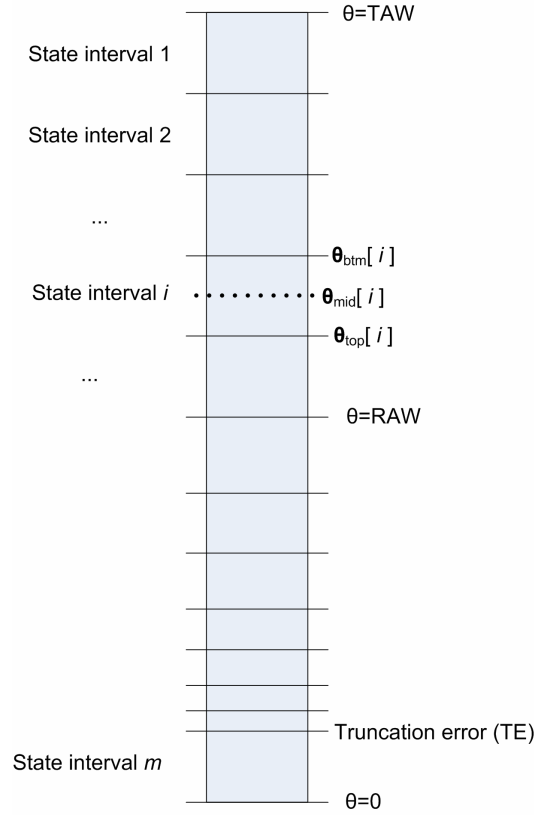
i_g = Irrigation depth vector (mm)

Equation 7-10: Mean water use.

An explanation of the irrigation depth vector (i_g) is presented alongside the irrigation PTM in Section 7.5.4.

7.5.4 Soil moisture state discretisation

Soil moisture was used as the state variable and was discretised such that the amount of depletion from actual ET over one time discretisation interval resulted in soil moisture decreasing by exactly one state interval. In the absence of rainfall or irrigation, the soil moisture reduces linearly until the soil moisture deficit is greater than the readily available water. Beyond this point, soil moisture reduces exponentially, asymptotically approaching the permanent wilting point. Since wilting point is never reached, if the difference between every sequential soil moisture state interval was equivalent to the depletion from ET, there would be an infinite number of state intervals. Therefore the last state interval has a small but finite upper limit (termed the *truncation error*) and wilting point as the lower limit. Soil moisture state discretisation is defined in Equation 7-11.



$$\theta_{\text{btm}}[i] = \begin{cases} \text{TAW} - (i-1)e_p & 1 \leq i \leq \frac{\text{RAW}}{e_p} + 1 \\ (\text{TAW} - \text{RAW}) \text{Exp}\left(\frac{\text{RAW} - (i-1)e_p}{\text{TAW} - \text{RAW}}\right) & \frac{\text{RAW}}{e_p} + 1 < i < m \end{cases} \quad [\text{a}]$$

$$\theta_{\text{top}}[i] = \begin{cases} \theta_{\text{top}}[i+1] & 1 \leq i < m \\ 0 & i = m \end{cases} \quad [\text{b}]$$

$$\theta_{\text{mid}}[i] = \frac{1}{2} (\theta_{\text{btm}}[i] + \theta_{\text{top}}[i]) \quad 1 \leq i \leq m \quad [\text{c}]$$

$$m = \frac{\text{RAW}}{e_p} + \frac{\text{TAW} - \text{RAW}}{e_p} \ln\left(\frac{\text{RAW} - \text{RAW}}{\text{TAW} \times \text{TE}}\right) + 1 \quad [\text{d}]$$

Where:

- $\theta_{\text{btm}}[m]$ = Soil moisture state discretisation vector – bottom of discretisation intervals (mm)
- $\theta_{\text{top}}[m]$ = Soil moisture state discretisation vector – top of discretisation intervals (mm)
- $\theta_{\text{mid}}[m]$ = Soil moisture state discretisation vector – mid-point of discretisation intervals (mm)
- m = Number of state discretisation intervals
- e_p = Potential evapotranspiration (mm/day)
- TAW = Total (plant) available water (mm)
- RAW = (Plant) readily available water (mm)
- TE = Truncation error (as a proportion of TAW)

Equation 7-11: State discretisation of soil moisture.

The soil moisture re-discretisation matrix describes the relationship between two different soil moisture state discretisation instances. Unlike a PTM, this matrix is not generally square. It is a sparse matrix, with all non-zero entries occurring in a band extending from the top left corner to the lower right corner. Prior to applying this matrix, soil moisture discretisation requires adjustment if there is a change in rooting depth. Root extension results in an increase in TAW, while root die-off a reduction in TAW. It was assumed that any new soil that roots extended into was at field capacity, and when root-die occurred, the reduction in TAW was proportional to the reduction in rooting depth. The influences of changes in rooting depth and the soil moisture re-discretisation matrix are defined in Equation 7-12. The soil moisture re-discretisation matrix is illustrated by example in Figure 7-3.

(1) Adjusting state discretisation in response to rooting depth changes:		
IF	$TAW_k < TAW_{k+1}$	$\theta_{btm_k} = \theta^*_{btm_k} + (TAW_{k+1} - TAW_k)$
ELSE IF	$TAW_k > TAW_{k+1}$	$\theta_{btm_k} = \theta^*_{btm_k} \frac{TAW_{k+1}}{TAW_k}$
ELSE		$\theta_{btm_k} = \theta^*_{btm_k}$
And		
$\theta_{top}[i] = \begin{cases} \theta_{btm}[i+1] & 1 \leq i < m-1 \\ 0 & i = m \end{cases} \quad (\text{Equation 7-11[b]})$		
(2) Re-discretisation matrix (applied after rooting depth adjustment):		
IF	$\theta_{btm_k}[i] \leq \theta_{btm_{k+1}}[j] \ \& \ \theta_{top_k}[i] \geq \theta_{top_{k+1}}[j]$	$\Theta[i,j] = 1.0$
ELSE IF	$\theta_{btm_k}[i] \geq \theta_{btm_{k+1}}[j] \ \& \ \theta_{top_k}[i] \leq \theta_{top_{k+1}}[j]$	$\Theta[i, j] = \frac{\theta_{btm_{k+1}}[j] - \theta_{top_{k+1}}[j]}{\theta_{btm_k}[i] - \theta_{top_k}[i]}$
ELSE IF	$\theta_{btm_k}[i] > \theta_{btm_{k+1}}[j] \ \& \ \theta_{top_{k+1}}[j] < \theta_{top_k}[i] < \theta_{btm_{k+1}}[j]$	$\Theta[i, j] = \frac{\theta_{btm_{k+1}}[j] - \theta_{top_k}[j]}{\theta_{btm_k}[i] - \theta_{top_k}[i]}$
ELSE IF	$\theta_{top_{k+1}}[j] < \theta_{btm_k}[i] < \theta_{btm_{k+1}}[j] \ \& \ \theta_{top_k}[i] < \theta_{top_{k+1}}[j]$	$\Theta[i, j] = \frac{\theta_{btm_k}[j] - \theta_{top_{k+1}}[j]}{\theta_{btm_k}[i] - \theta_{top_k}[i]}$
ELSE		$\Theta[i,j] = 0.0$
Where:		
$\Theta[m_k, m_{k+1}]$ = Soil moisture re-discretisation matrix, where dimensions m_x are the number of state discretisation intervals for soil moisture discretisation on day x		
θ^*_{btm} = Soil moisture state discretisation vectors prior to rooting depth adjustment		
$\{\theta_{top_x}, \theta_{btm_x}\}$ = Soil moisture state discretisation vectors (top & bottom respectively) on day x		

Equation 7-12: Rooting depth changes and soil moisture re-discretisation matrix.

$$\Theta = \begin{pmatrix} 0.87 & 0.13 & 0 & 0 & .. & & & & & & & & & & \\ 0 & 0.74 & 0.26 & 0 & 0 & .. & & & & & & & & & \\ 0 & 0 & 0.60 & 0.40 & 0 & 0 & .. & & & & & & & & \\ .. & 0 & 0 & 0.44 & 0.56 & 0 & 0 & .. & & & & & & & \\ & .. & 0 & 0 & 0.37 & 0.63 & 0 & 0 & .. & & & & & & \\ & & .. & 0 & 0 & 0.23 & 0.77 & 0 & 0 & .. & & & & & \\ & & & .. & 0 & 0 & 0.09 & 0.87 & 0.05 & 0 & 0 & .. & & & \\ & & & & .. & 0 & 0 & 0 & 0.83 & 0.17 & 0 & 0 & .. & & \\ & & & & & .. & 0 & 0 & 0 & 0.71 & 0.29 & 0 & 0 & & \\ & & & & & & .. & 0 & 0 & 0 & 0.58 & 0.42 & 0 & & \\ & & & & & & & .. & 0 & 0 & 0.10 & 0.90 & & & \end{pmatrix}$$

Figure 7-3: Example of a soil moisture re-discretisation matrix.

7.5.5 Irrigation

The irrigation PTM (\mathbf{I}_g) describes the change in the soil moisture PV (θ_{pv}) in response to irrigation. Irrigation is assumed to occur if the soil moisture is below a specified TSML. The matrix is sparse, all elements are either 1 or 0, and lower triangular (since irrigation never decreases soil moisture). The irrigation depth vector (\mathbf{i}_g) is a vector description of the TSML. It specifies that when the soil moisture is above the TSML the depth of applied irrigation water is zero, and when it is below the TSML the depth of applied water is approximately equal to the application depth multiplied by the application efficiency. However, to avoid ‘numerical dispersion’¹⁵, the amount of water applied (via \mathbf{i}_g) is not always exactly equal to the application depth multiplied by the application efficiency (i.e. Equation 7-13[b]). Both \mathbf{I}_g and \mathbf{i}_g are presented in Equation 7-13 and illustrated by example in Figure 7-4.

¹⁵ Since the probability distribution in any interval is assumed to be uniform before and after the application of a PTM, when the change in θ is not exactly equal to the mid-point of the interval, over-successive applications of the PTM errors will accumulate.

<u>Condition</u>		$\mathbf{I}_g[i,j]$	$i_g[i]$	<u>Action/Comment</u>
IF	$\theta_{mid}[i] > t_{sml} \text{ TAW}$ & $i = j$	1.0	0	No irrigation.
ELSE IF	$\theta_{mid}[i] < t_{sml} \text{ TAW}$ & $(\theta_{mid}[j] - \theta_{mid}[i]) \sim q D$ & $t_{sml} \text{ TAW} < \text{TAW} - q D$	1.0	$\theta_{mid}[j] - \theta_{mid}[i]$	Irrigate. $i_g[i]$ adjusted to avoid numerical dispersion. If UCC = 1.0, no drainage occurs.
ELSE IF	$\theta_{mid}[i] < t_{sml} \text{ TAW}$ & $(\theta_{mid}[j] - \theta_{mid}[i]) \sim q D$ & $t_{sml} \text{ TAW} > \text{TAW} - q D$	1.0	$q D$	Irrigate. No adjustment to $i_g[i]$. If UCC = 1.0, drainage occurs.
ELSE		0.0	NA	None

Where:

$\mathbf{I}_g[m,m]$ = Irrigation PTM (unitless)

$i_g[m]$ = Irrigation depth vector (mm)

θ_{mid} = Soil moisture state discretisation vector – mid-point of discretisation intervals (Equation 7-11)

TAW = Total (plant) available water (mm)

t_{sml} = Trigger soil moisture level (portion of TAW)
= $\mathbf{T}_{sml}[a,b]$ where a and b are the given TAP and Block, respectively (Table 6-1)

q = Application efficiency (Equation 2.1)

D = Mean irrigation infiltration depth (mm)

m = Number of state discretisation intervals

Equation 7-13: Irrigation PTM and depth vector.

$$\mathbf{I}_g = \begin{pmatrix} 1 & 0 & 0 & .. \\ 0 & 1 & 0 & 0 & .. \\ 0 & 0 & 1 & 0 & 0 & .. \\ .. & 0 & 0 & 1 & 0 & 0 & .. \\ .. & 0 & 0 & 1 & 0 & 0 & .. \\ . & 0 & 0 & 1 & 0 & 0 & .. \\ 0 & 1 & 0 & 0 & .. & .. \\ 0 & 0 & 1 & 0 & 0 & .. \\ 0 & 0 & 1 & 0 & 0 & .. \\ .. & 0 & 0 & 1 & 0 & 0 & .. \end{pmatrix} \quad \mathbf{i}_g = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 39.1 \\ 41.2 \\ 39.7 \\ 40.0 \end{pmatrix}$$

Figure 7-4: Example of irrigation PTM and depth vector ($q D = 40$ mm).

A constraint on system capacity is included into \mathbf{I}_g and i_g by checking the predicted water use (Equation 7-10) over some time period (typically 1 month), to see if the predicted water use is greater than the system capacity. If water use exceeds the system capacity, the TSML is iteratively reduced until the predicted water use is equal to the system capacity. In practice, this would mean that irrigation actually occurs at a lower trigger level (because of irrigation system limitations), while the goal may be to irrigate at a higher specified trigger level.

The ET PTM (\mathbf{E}_a) describes the change in the soil moisture PV in response to ET. Potential ET is assumed to be deterministic (varying as a function of the time of year), and actual ET is modelled as per Equation 7-2. The effect of the water stress coefficient is directly included into the formula of \mathbf{E}_a and the discretisation of the soil moisture (Equation 7-11). The soil moisture state discretisation has been specifically formulated such that the daily reduction in soil moisture from ET exactly reduces soil moisture by one state interval. The ET PTM is defined in Equation 7-14 and illustrated by example in Figure 7-5.

Equation 7-14: ET PTM.

Figure 7-5: Example of actual ET PTM.

7.5.7 Effective precipitation

The precipitation PTM (U_e) describes the change in soil moisture PV in response to rainfall. Rainfall was assumed to have a Poisson storm arrival and Gamma depth distribution, as described in Equation 3-3. Since one or more independent storm events may occur per day, the PDF of the wet day precipitation depth (PDF_{uw}) is the sum of an infinite series of Gamma PDFs (Equation 3-5[f]), while the probability of a wet day is given by Equation 3-4[a].

Two approximations of the above rainfall distribution are included into U_e – one explicitly and one implicitly. The explicit approximation made was that λ is small. As $\lambda \rightarrow 0$, Equation 3-4[a] and Equation 3-5[f] simplify to Equation 7-15. The main advantage of this approximation is that all elements in U_e have an analytical solution, thereby minimising cumulative numerical errors. Importantly, this approximation does not change mean daily rainfall (wet day probability multiplied by the mean wet day depth). The implicit approximation is associated with the Markov chain assumption that the soil moisture within a state interval has a uniform probability distribution prior to applying a PTM. The implicit effect of this assumption is that the wet day precipitation depth PDF is approximated with a step-wise function. Again, this approximation does not alter mean daily rainfall. The error introduced by both approximations tends to zero, as the time discretisation interval (Δt) tends to zero. These approximations are illustrated in Figure 7-6. Climate parameters used are for January at Christchurch Airport (Table 3-4).

$P[\text{Rain during 1 day}] = \lambda$	[a]
And	
$PDF_{uw}(u_w) = \frac{(\alpha_s^{\beta_s} e^{-\alpha_s u_w}) u_w^{\beta_s - 1}}{\Gamma(\beta_s)}$	[b]
Where:	
$PDF_{uw}(u_w)$ = PDF of the wet day precipitation depth	
λ = Mean number of storm events per day	
u_w = Wet day precipitation depth (mm)	
α_s = Gamma scale parameter for storm depths (mm^{-1})	
β_s = Gamma shape parameter for storm depths	

Equation 7-15: Daily rainfall probability as the mean number of storm events per day approaches zero.

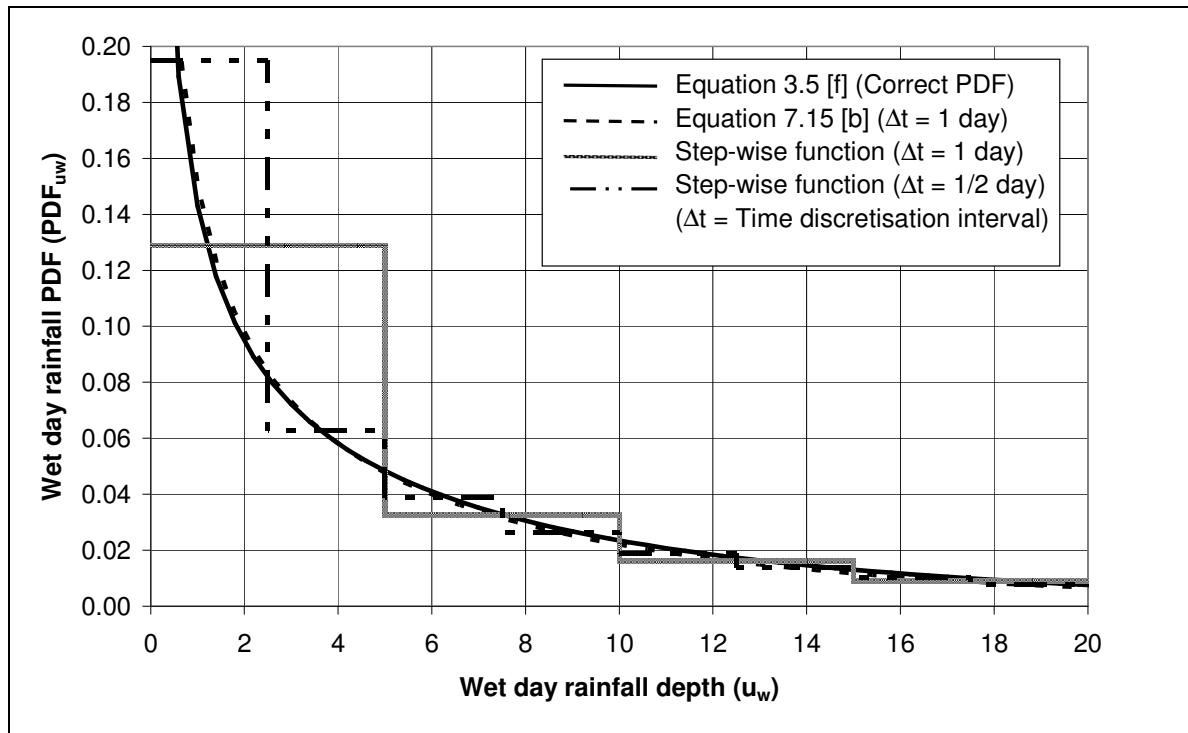


Figure 7-6: Approximating the PDF of the wet day precipitation depth; using climate parameters from Christchurch Airport, for January.

From Figure 7-6, it can be seen that the more significant of the two approximations is the implicit approximation of the PDF distribution with a step-wise function. Approximating the shape of the wet day precipitation depth PDF only is significant when drainage is significant. For well-managed irrigation, drainage will primarily occur during the shoulders of an irrigation season. However, at these times potential ET is low, which will decrease the width of state discretisation intervals and improve the approximation of the PDF by reducing the step width (since the width of each step in the step-wise function is equal to the potential ET multiplied by the time discretisation interval). Nevertheless, for irrigation regimes that promote drainage, approximating the shape of the PDF with a step-wise function will slightly increase predicted water use. Errors introduced by these approximations were quantified in Section 4.5.8 and, as expected, were shown to be relatively small.

The PTM for effective precipitation is a lower triangular matrix, since rain never decreases soil moisture. The matrix is defined in Equation 7-16 and illustrated by example in Figure 7-7 (Christchurch in January with TAW = 40 mm).

$U_e = \lambda W + (1-\lambda)I$		
And		
IF $\{i, j\} \leq m \& j \leq i$		
IF	$i = j = 1$	$W[i,j]=1.0$
ELSE IF	$j = 1 \& i = 2$	$W[i, j] = 1 + f_1(C_4) - f_1(0) - f_2(C_4)$
ELSE IF	$j = 1$	$W[i, j] = 1 + f_1(C_4) - f_1(C_1) - f_2(C_4)$
ELSE IF	$i = j > 1$	$W[i, j] = f_3(C_2) - f_3(0)$
ELSE IF	$C_5 \geq C_2 \& i = j + 1$	$W[i, j] = f_1(C_4) - f_1(0) + f_2(C_6) - f_2(C_4) + f_3(C_3) - f_3(C_6)$
ELSE IF	$C_5 \geq C_2$	$W[i, j] = f_1(C_4) - f_1(C_1) + f_2(C_6) - f_2(C_4) + f_3(C_3) - f_3(C_6)$
ELSE IF	$i = j + 1$	$W[i, j] = f_1(C_6) - f_1(0) + \frac{C_5(f_2(C_4) - f_2(C_6))}{C_2} + f_3(-C_3) - f_3(C_4)$
ELSE		$W[i, j] = f_1(C_6) - f_1(C_1) + \frac{C_5(f_2(C_4) - f_2(C_6))}{C_2} + f_3(-C_3) - f_3(C_4)$
ELSE $W[i,j]=0.0$		
Where:		
$f_1(z) = \frac{\alpha C_1 \Gamma(\beta, \alpha z) - \Gamma(\beta + 1, \alpha z)}{\alpha C_2 \Gamma(\beta)}$		
$f_2(z) = 1 - \frac{\Gamma(\beta, \alpha z)}{\Gamma(\beta)} = \text{Gamma distribution cumulative density function}$		
$f_3(z) = \frac{\alpha C_3 \Gamma(\beta, \alpha z) + \Gamma(\beta + 1, \alpha z)}{\alpha C_2 \Gamma(\beta)}$		
In the limit as $z \rightarrow 0$:		
$f_1(z \rightarrow 0) = \frac{(\alpha C_1 - \beta)}{\alpha C_2}, f_2(z \rightarrow 0) = 0, f_3(z \rightarrow 0) = \frac{\beta + \alpha C_3}{\alpha C_2}$		
C_1	$= \theta_{\text{top}}[j] - \theta_{\text{btm}}[i]$	$C_4 = \theta_{\text{top}}[j] - \theta_{\text{top}}[i]$
C_2	$= \theta_{\text{btm}}[i] - \theta_{\text{top}}[i]$	$C_5 = \theta_{\text{btm}}[j] - \theta_{\text{top}}[j]$
C_3	$= \theta_{\text{btm}}[j] - \theta_{\text{top}}[i]$	$C_6 = \theta_{\text{btm}}[j] - \theta_{\text{btm}}[i]$
$U_e[m,m]$ = Precipitation Probability Transition Matrix (unitless)		
$W[m,m]$ = Wet day Probability Transition Matrix (unitless)		
$I[m,m]$ = Identity matrix (unitless)		
λ = Mean number of storm events per day		
α = Gamma scale parameter for storm depths (mm^{-1}) ($= \alpha_s$)		
β = Gamma shape parameter for storm depths (unitless) ($= \beta_s$)		
m = Number of state discretisation intervals		
$\Gamma(z) = \int_0^\infty t^{z-1} \text{Exp}(-t) dt = \text{Euler's Gamma function}$		
$\Gamma(a,z) = \int_z^\infty t^{a-1} e^{-t} dt = \text{Incomplete Gamma function}$		

Equation 7-16: Effective precipitation PTM.

$$U_e = \begin{pmatrix} 1.00 & 0 & 0 & .. & & & & & & & \\ 0.11 & 0.89 & 0 & 0 & .. & & & & & & \\ 0.06 & 0.06 & 0.89 & 0 & 0 & .. & & & & & \\ 0.03 & 0.02 & 0.06 & 0.89 & 0 & 0 & .. & & & & \\ 0.02 & 0.01 & 0.03 & 0.06 & 0.88 & 0 & 0 & .. & & & \\ 0.01 & 0.01 & 0.01 & 0.03 & 0.06 & 0.87 & 0 & 0 & .. & & \\ 0.01 & 0.01 & 0.01 & 0.02 & 0.03 & 0.06 & 0.86 & 0 & 0 & .. & \\ 0.01 & 0.00 & 0.01 & 0.01 & 0.02 & 0.03 & 0.06 & 0.86 & 0 & 0 & .. \\ 0.01 & 0.00 & 0.01 & 0.01 & 0.02 & 0.02 & 0.03 & 0.05 & 0.85 & 0 & 0 \\ 0.01 & 0.00 & 0.01 & 0.01 & 0.01 & 0.02 & 0.02 & 0.03 & 0.05 & 0.84 & 0 \\ 0.00 & 0.00 & 0.00 & 0.01 & 0.01 & 0.01 & 0.02 & 0.02 & 0.02 & 0.02 & 0.88 \end{pmatrix}$$

Figure 7-7: Example of effective precipitation PTM

7.5.8 Testing via simulation

Several simulation studies were undertaken. The first objective was to illustrate that as the time discretisation interval and truncation error tends to zero (i.e. continuous time), the Markov Chain Water Use Equation is an exact solution of the mean irrigation water use, given a Type 1 water use model. The second objective was to assess suitable values for the truncation error and the time discretisation interval to maximise efficiency, while ensuring errors introduced by these two approximations were acceptable.

7.5.8.1 Continuous time domain

Table 7-6 shows the parameters for the simulation study when time discretisation interval (Δt) and truncation error (TE) tend to zero. Results are presented in Figure 7-8. Two different variants of the daily time-step Type 1 water use simulation model were used. The first simulation model (variant 1) assumed irrigation occurred just before rainfall (when both occurred on the same day). The second model variant (variant 2) assumed irrigation occurred just after any rainfall. The reason for these two variants is that for the Markov chain representation, as $\Delta t \rightarrow 0$ and $TE \rightarrow 0$, the time domain is continuous and events (such as irrigation and rainfall) can occur any time during the day. In contrast for simulation, time is discretised; therefore the order of irrigation and rainfall does affect the predicted water use. This is principally because knowledge of whether rain will occur on a given day may delay an irrigation event. The solution for a simulation model, which was continuous in time, would lie between the solutions of these two model variants. Both simulation variants were modified so that the water stress coefficient was based on the soil moisture when half of the ET had been removed. This more closely models ET removal occurring continuously throughout the day, rather than instantaneously at the start of the day.

Table 7-6: Simulation study parameters: Comparing the Markov Chain Water Use Equation as ($\Delta t \rightarrow 0$, $TE \rightarrow 0$), with simulation of multiple seasons.

Parameter	Value
Model description	Single soil layer, single crop coefficient FAO 56 crop model after Allen et al. (1998)
Crop	Pasture – constant rooting depth and crop coefficient of 1.0
Soil TAW	80 mm
Soil RAW	50% of TAW
Irrigation depth	20 mm
System capacity	5 mm/day (Minimum return period 1 day in 4)
Trigger soil moisture level	Variable
Irrigation season	1 September – 31 March
UCC	1.0
Case study irrigation years	1000 years of synthetically generated (seasonally average mean ETo, compound-Poisson rainfall) climate, derived from Christchurch historic climate (Table 3-1)

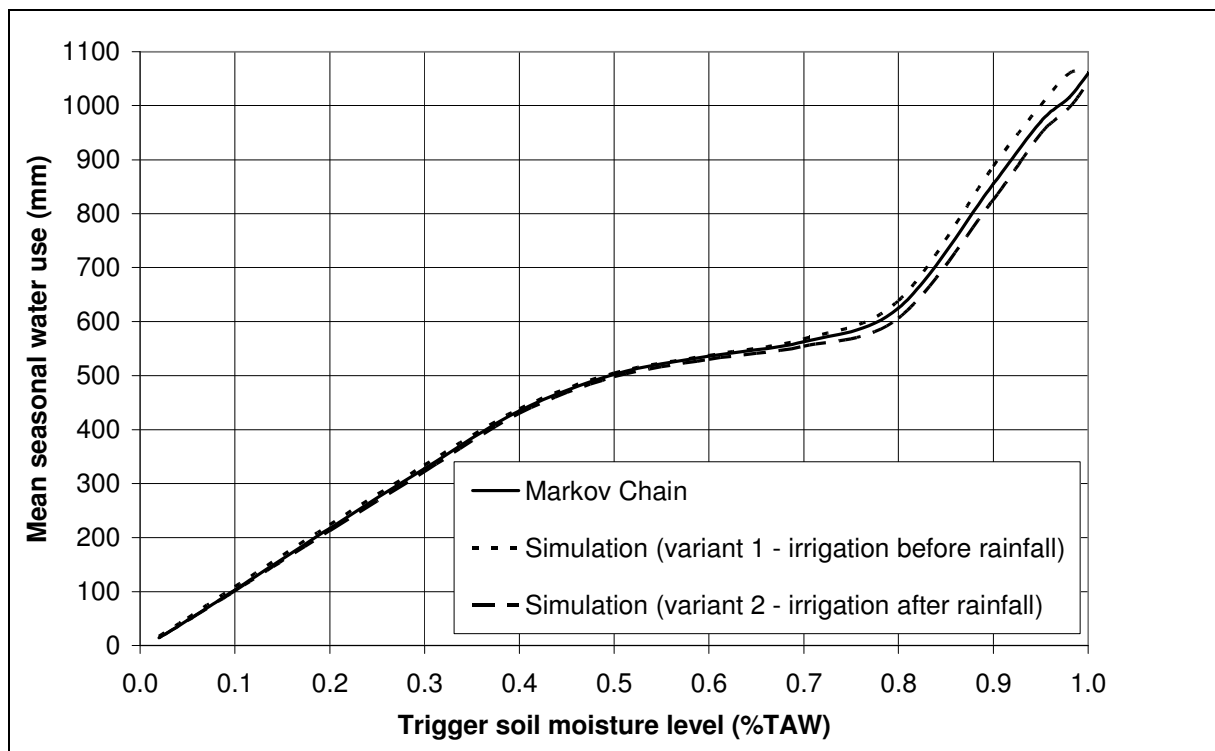


Figure 7-8: Comparing the Markov Chain Water Use Equation as ($\Delta t \rightarrow 0$, $TE \rightarrow 0$), with simulation of multiple seasons, assuming a Type 1 water use model.

Figure 7-8 shows that the Markov chain equation (for $\Delta t \rightarrow 0$ and $TE \rightarrow 0$) consistently gave results that lie between the two simulation variants, illustrating that the equation is an exact solution of a Type 1 water use model (Table 7-1) as the time discretisation interval and truncation error tend to zero.

7.5.8.2 Truncation Error

The truncation error (TE) is the upper boundary of the last state interval, expressed as a proportion of TAW (Section 7.5.4). Increasing the TE increases computational efficiency (through decreasing the number of state intervals – Equation 7-11[d]) but decreases accuracy. A study was undertaken to determine an appropriate level for the TE that would provide a trade-off between efficiency and accuracy. Case study parameters are as per Table 7-6, with the exception that the irrigation season extends from 1 September to 14 April, and the system capacity is 8 mm/day (unconstraining). Results are presented in Figure 7-9.

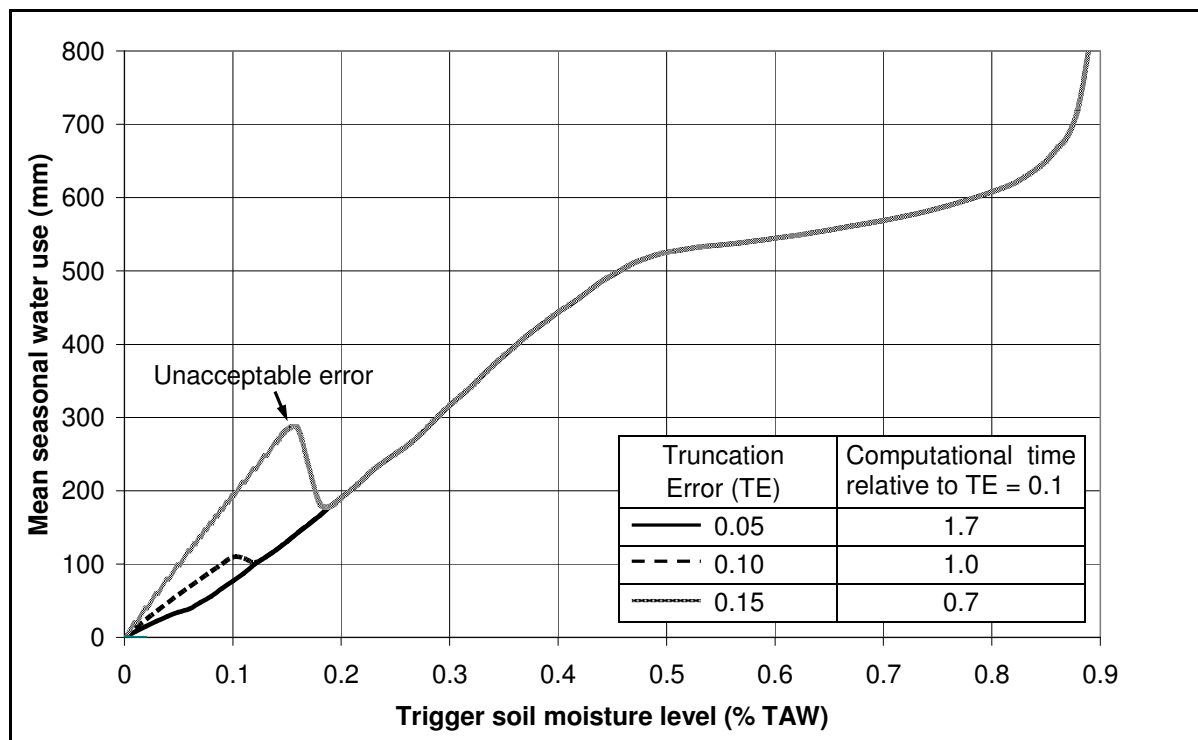


Figure 7-9: Truncation error: Trade-off between accuracy and computation requirements.

It was concluded from the above analysis that for most cases, a TE value of 0.10 is an appropriate trade-off between accuracy and efficiency requirements.

7.5.8.3 Discrete time domain

It is impractical for $\Delta t \rightarrow 0$ and $t_e \rightarrow 0$ since computational demands are excessive. Cordova and Bras (1979) suggested a value of $\Delta t = 1$ day was a reasonable trade-off between mathematical accuracy and efficiency. An argument for TE of 0.1 was given in the previous section. A study was undertaken to compare the Markov chain equation with continuous time ($\Delta t \rightarrow 0$, $t_e \rightarrow 0$) and discrete time ($\Delta t = 1$ day, TE = 0.1). The study covered a range of trigger soil moisture levels and a wide range of possible soil TAW. Parameters are as per Table 2-4, with the exception of variations in TAW. Results are shown in Figure 7-10.

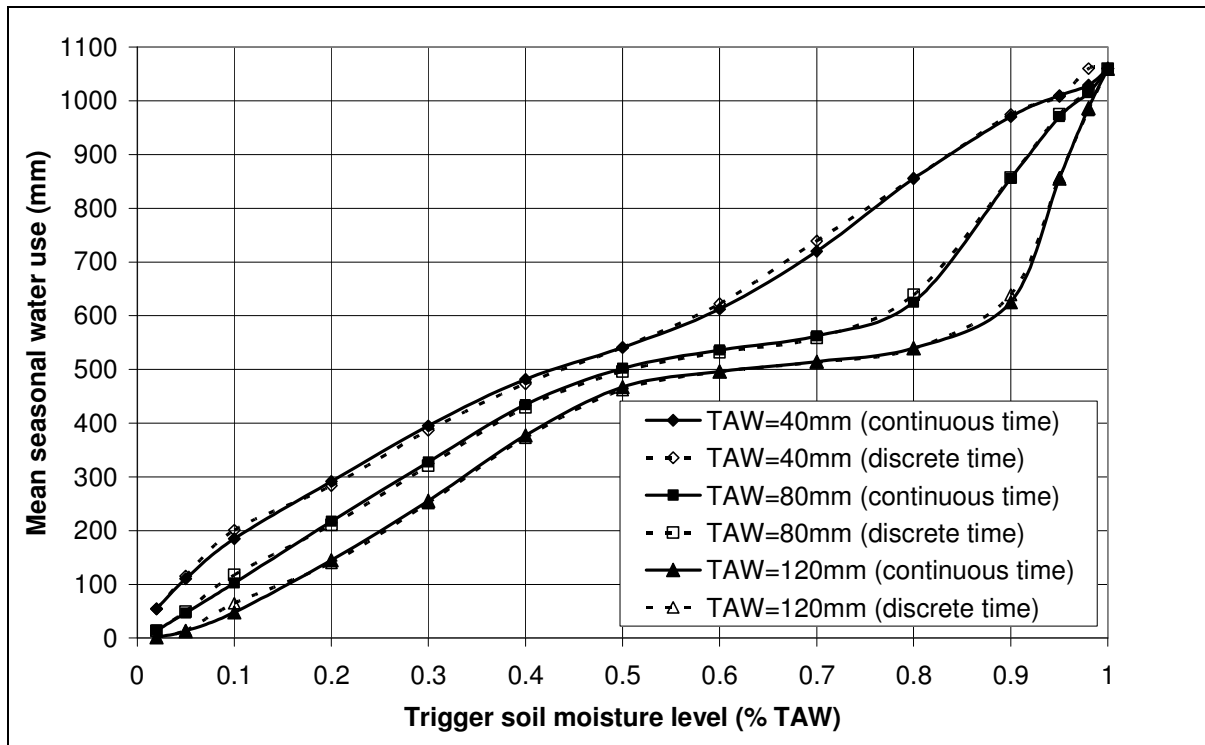


Figure 7-10: Comparison of Markov chain equation for continuous and discrete ($\Delta t = 1$ day and TE of 0.1) time.

Results from Figure 7-10 show errors introduced by time discretisation (for $\Delta t = 1$, TE of 0.1) were small (generally less than 3%).

7.5.9 Implementation issues

Equation 7-9 and Equation 7-10 can be implemented highly efficiently by assuming climate, soil, irrigation and plant properties do not change within a given TAP, and by using pre-calculation and storage. In the research and development code implementation, estimation of mean seasonal water use for a season takes approximately 0.4 ms on a P4 2.8 GHz computer. However, theoretically, this time could be reduced by one order of magnitude by improving code efficiency such that the majority of the computational demand to associate with matrix-matrix multiplications. Furthermore, due to the way the Markov chain equation is implemented, it is possible to decrease the number of soil moisture state discretisation intervals (m) in exchange for a reduction in accuracy. The computational demand for matrix-matrix multiplication is m to the power of three.

7.5.10 Incorporating Type 2 and Type 3 irrigation water use models

It is possible to extend the above Markov chain equation to include a Type 2 irrigation water use model (Section 7.2). The principal limitation is the increased computation demand. The matrix-matrix multiplication equivalent operation for a dual layer soil moisture model requires multiplying a four-dimensional array with a computational demand equal to the number of soil moisture state discretisation intervals (m) to the power of seven – compared with m to the power of three for the

single layer model. Due to the way the Markov chain equation is implemented, it is possible to decrease m in exchange for a reduction in accuracy. Such a trade-off could result in a sufficiently small value of m , such that the computational demand of the dual soil layer Markov chain equation is about 50 times that of the single soil-layer variant. With an efficient computer implementation, this would correspond to a computation time (for estimating water use for a season) in the order of 1 ms on a P4 2.8 GHz computer. This extension is an area requiring further research.

The extension to a Type 3 irrigation water use model may be possible through the incorporation of evaporation adjusted single crop coefficient model. One issue with this extension is that the change in potential ET results in a change in soil moisture discretisation. Due to the way the Markov chains are implemented (through minimising repeating calculations through the use of memory), each additional soil moisture discretisation instance results in a corresponding increase in memory requirements. Further research is required to ascertain whether or not memory requirements would be constraining.

The Markov chain method can be extended to allow for irrigation modifying soil moisture in a spatially probabilistic manner (due to spatial non-uniform application). This extension is incorporated by modifying both the irrigation PTM and depth vector (Equation 7-13), such that the soil moisture response to an irrigation event is probabilistic rather than deterministic. This extension is an area requiring further research.

7.6 Up scaling to Water Use for a Block

The majority of the preceding chapter has assumed water use is for a single Paddock. However, the SA scheduler requires water use estimates for Blocks (\mathbf{A}_B). The difference between estimating water use for a Paddock and for a Block is that the initial soil moisture for a Paddock is a single value, while the soil moisture for a Block is spatially probabilistic (since soil moisture will vary from Paddock to Paddock). Incorporating this initial probabilistic soil moisture into the Markov Chain Water Use Equation is very simple, since the soil moisture PV already allows for the initial soil moisture to be probabilistic (Equation 7-10). Therefore, the water use equation given in Equation 7-10 is valid for estimating Block water use without modification.

In order for the Simulation or DPD Water Use Equations to incorporate initial probabilistic soil moisture, these equations would need to be used multiple times, each time with different initial soil moisture. Provided the time period for which water use is required is sufficiently long, by taking the initial soil moisture as the mean of the soil moisture in all Paddocks within the Block and using these equations only once, the error introduced may be acceptably small. Further work is required to quantify under what circumstances the probabilistic initial soil moisture for a Block may be approximated by the mean value, and under what circumstances multiple different initial soil moisture status are required.

7.7 Inverse Water Use Equation

Most of this chapter has focused on water use as a function of trigger soil moisture levels: $A_B = f_{WUE}(T_{sml})$. The Inverse Water Use Equation is an equation relating trigger soil moisture levels as a function of water use: $T_{sml} = f^{-1}_{WUE}(A_B)$ (Equation 6-2). Both equations are used within the SA scheduler, with the latter being used more frequently than the former. For all Water Use Equation methods (Simulation, DPD, Markov chains), the inverse equation may be calculated by using f_{WUE} and iteratively adjusting $A_B[i \text{ TAP}, j \text{ Block}]$ until $T_{sml}[i \text{ Tap}, j \text{ Block}]$ is the desired value. Ridders' root finding method was used. This root-finding method will be less efficient if the Simulation Water Use Equation is used, due to discontinuities in this equation. Where the Water Use Equation is smooth (DPD or Markov chain methods), Ridders' root finding procedure was able to converge to about three significant figure accuracy within four to five iterations.

7.8 Applications Outside of the SA scheduler

The development of the Water Use Equations within this chapter has been focused exclusively on their application within the SA scheduler. Outside the scope of this thesis, simulation is the established technique for estimating water use. The alternative methods for calculating water use, Markov chains and DPD, are novel, and there may be other applications for these approaches. The main strength of the Markov chain approach is the ability to take into account a number of stochastic variables in an efficient manner (providing an alternative to Monte-Carlo Simulations). An example of a possible application could be quantifying how spatial non-uniformity (in applied nitrogen, in the form of cow urine patches) influences the amount of nitrogen leached. The DPD method was surprisingly successful, given the required assumptions. For use within the SA scheduler, this method was superseded by the Markov chain method. However, lessons learned in deriving the DPD equation were used in the development of the Markov chain method, and it is possible that there may be further benefits derived from a hybrid of these two methods.

7.9 Conclusions

Three different irrigation water use model types for modelling water use were defined in Table 7-1. While a Type 3 model would be ideal, the impact on the SA scheduler's performance from incorporating simpler models within the Water Use Equation (e.g. Type 1) has not been assessed.

The standard method for predicting irrigation water use is via a farm simulation. This method can easily incorporate Type 1 to Type 3 water use models, with minimal differences in the computational demands. The ability to incorporate historic climate is dependent on the number of years of available data and the required level of accuracy, which is determined by the number of seasons simulated. It is not possible to include spatial soil variability without a large increase in computational demand. The two disadvantages of using simulation for Type 1 to Type 3 irrigation water use models are (a) the

computational demand is high, and (b) the relationship between the TSML and mean water use is not smooth.

The DPD Water Use Equation assumes a Type 1 irrigation water use model but with the exclusion of Bright's application uniformity model. This method assumes that the soil moisture PDF is independent of the timing and depth of specific rain events, which is reasonable during the peak of the season (November to February) but less valid during the shoulders of the season (September-October and March-April). This method was not extended to include Type 2 or Type 3 irrigation water use models.

The principal advantage of the Markov Chain Water Use Equation over the DPD method is that it is not necessary to make the assumption that the soil moisture PDF is independent of the timing and depth of specific rain events. This Markov Chain Water Use Equation is an exact solution of the mean irrigation water use, given a Type 1 irrigation water use model. The Markov chain method also has potential to be extended to Type 2 and Type 3 irrigation water use models. The computational demand for a Type 1 irrigation water use model is 2-3 orders of magnitude more efficient than calculating mean water use by simulation. This method has the added advantage that it can easily be extended to allow for irrigation modifying soil moisture in a spatially probabilistic manner (due to spatial non-uniform application), and that the calculation demand of water use for a Block is not greater than for a single Paddock.

In conclusion, further work is required to definitively state under what circumstances simulation or Markov chains is the most appropriate method for use within the SA scheduler. Current indications are that the Markov chain method (extended to Type 2 and Type 3 irrigation water use models) will offer greater advantages than calculating mean water use by simulation.

The current implementation of the SA scheduler (used in the case studies in Chapters 9 and 10) uses the Markov Chain Water Use Equation with a Type 1 irrigation water use model.

8 HEURISTIC METHOD SELECTION AND DESIGN

8.1 Context and Overview

This chapter develops the heuristic method used in the SA scheduler described in Chapter 6. An original optimisation methodology was required due to the complexity of the optimisation problem.

In Chapter 5, the comment was made that for low dimensionality unconstrained optimisation problems, ‘off the shelf’ heuristics may be applied. However, as the dimensionality of the problem increases, it is more likely that a custom heuristic method will be required. Chapter 6 concluded that optimisation for the SA scheduler would require a custom heuristic, due to the problem dimensionality and constraints.

From Chapter 6, the dependence¹⁶ between decision variables suggested that the optimisation problem was best described in continuous space, allowing gradient information to be utilised. Press et al.’s (2002) simulated annealing (SA) for continuous variables was a proposed method that could utilise this gradient information. This chapter makes two main extensions to Press et al.’s method: the incorporation of equality constraints, and the utilisation of population information. SA using discrete space was also implemented but, as predicted, was found to be inferior to optimisation in continuous space. Brief consideration is also given to alternative heuristic methods.

Ideas from this chapter – in particular, incorporating linear equality constraints and population information into continuous variable SA – may be useful in other optimisation problems that have a similar structure. Further details of the algorithms developed may be found in the electronic version of the source code (Appendix 1).

8.2 Press et al.’s Simulated Annealing for Continuous Variables

The SA method for continuous variables by Press et al. (2002) is a modification of Nelder and Mead’s downhill simplex method (Section 5.3.3). It had the following advantages for use within the SA scheduler (Chapter 6):

- a) Optimisation is in continuous space, taking full advantage of solution space gradients.
- b) It is robust for extreme gradients and discontinuities. In particular, this allows inequality boundary conditions to be easily incorporated via large penalty terms.

¹⁶ Dependence in this context means the objective function performance given a particular decision variable value, is dependent on the value of other decision variables.

- c) Greater exploration occurs in dimensions where decision variables are contentious and less exploration of non-contentious variables, since the simplex automatically contracts in the dimensions where decision variables within a narrow range generally result in better objective function performance.
- d) It is easy to implement, and the underlying mechanism of optimisation is relatively simple.
- e) The simplex dimensions automatically adjust to be appropriate for the boundary conditions.
- f) The region inside the simplex (rather than a single trajectory) is sampled with each simplex move.
- g) Solution space searching is relatively unbiased.

The principal limitation of Press et al.'s existing method (for use in the SA scheduler) was it did not allow for the incorporation of equality constraints (Equation 6-1, [f] and [g]). Equality constraints mathematically represent a reduction in the degrees of freedom. Therefore, to not use these constraints to reduce dimensionality represents a missed opportunity. Furthermore, it is highly inefficient to deal with equality constraints via penalty functions, since this results in an attempt to optimise along the floor of a very narrow canyon, causing the optimiser to spend the majority of time seeking to satisfy these constraints (rather than optimising). Hence, a method for incorporating equality constraints into continuous variable SA method is presented in Section 8.3.1

8.3 Incorporating Constraints

8.3.1 Equality constraints

In Chapter 6, equality constraints [f] and [g] in Equation 6-1 were deliberately chosen to be linear. For linear equality constraints, provided all the starting points of Nelder Mead's simplex satisfy these equalities, all subsequent simplex moves will also satisfy these equalities. This occurs because all new simplex steps are always linear combinations of existing vertices. This geometric effect is illustrated in Figure 8-1 in three dimensions for the example $\sum \tau[i]=2.0$, and in the absence of any inequality constraints. The initial simplex is in the plane $\tau[1] + \tau[2] + \tau[3] = 2.0$. Consequently, all subsequent moves of the simplex remain in this same plane. As a result, the solution space has been reduced from three to two dimensions. In general, dimensionality is reduced by the number of equality constraints, as are the number of vertex of the simplex. In Figure 8-1, the simplex has three vertexes instead of four, which would be required without the equality constraint. Furthermore, no boundary penalty terms are required to enforce the constraint. By themselves, equality constraints do not cause preferential searching of the solution space (bias). However, when both equality and inequality constraints exist, some bias is introduced (Section 8.3.2). Bias (resulting from constraints rather than the solution space topography) is undesirable if it makes it difficult to explore promising regions of the solution space. A small amount of bias is unlikely to affect the performance of the optimiser, and may even slightly enhance performance if it increases the probability of searching adjacent to boundaries. However, significant amounts of bias have the potential to create regions of the solution space which have a very low probability of being explored.

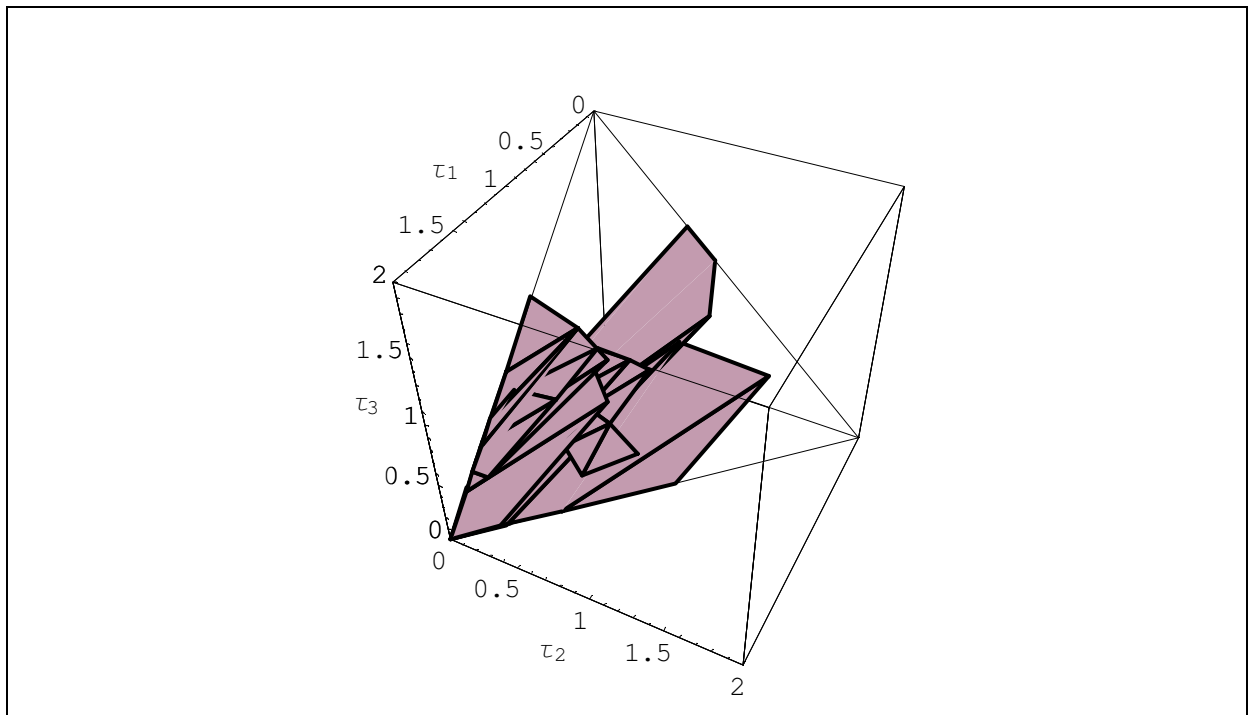


Figure 8-1: Equality constraint illustrated by random Nelder Mead simplex moves within the plane of $\tau[1] + \tau[2] + \tau[3] = 2.0$.

8.3.2 Inequality constraints

Equation 6-1 inequality constraints [d] and [e] are dealt with using penalty terms to discourage the simplex from traversing these boundaries. A valuable feature of Press et al.'s method of SA for continuous variables is the simplex size automatically adjusts such that it has the appropriate width in each dimension, for any boundary constraints. This occurs since each contact with the boundary causes the simplex to contract, while in the absence of contacting the boundary and at high annealing temperatures the simplex has a tendency to enlarge. At high annealing temperatures (when the topography of the feasible solution space has a negligible effect on the simplex behaviour) approximately 30% of simplex moves resulted in contact with the boundary¹⁷. This percentage appeared to be independent of the number of decision variables. This percentage decreases greatly at lower annealing temperatures since the feasible solution space topography does affect the behaviour of the simplex, reducing the simplex size and constraining the simplex to certain regions of the solution space.

Figure 8-2 illustrates the effect of inequality constraints on sampling bias, by comparing the Cumulative Density Function (CDF) of decision variables x_i (where x_i is a surrogate for Equation 6-1 decision variables $\tau[i]$ or $\mathbf{B}[i,j]$) for high temperature continuous variable SA, with unbiased sampling. The CDF for unbiased sampling was obtained by discretising x_i and generating all possible solutions. For this example, the equality constraint is $x_1 + x_2 + x_3 + \dots + x_7 = 0.6$ and the inequality constraints are $0 \leq x_i \leq 0.6$.

¹⁷ Given seven decision variables, $0 \leq x_i \leq 0.6$, and $x_1 + x_2 + x_3 + \dots + x_7 = 0.6$.

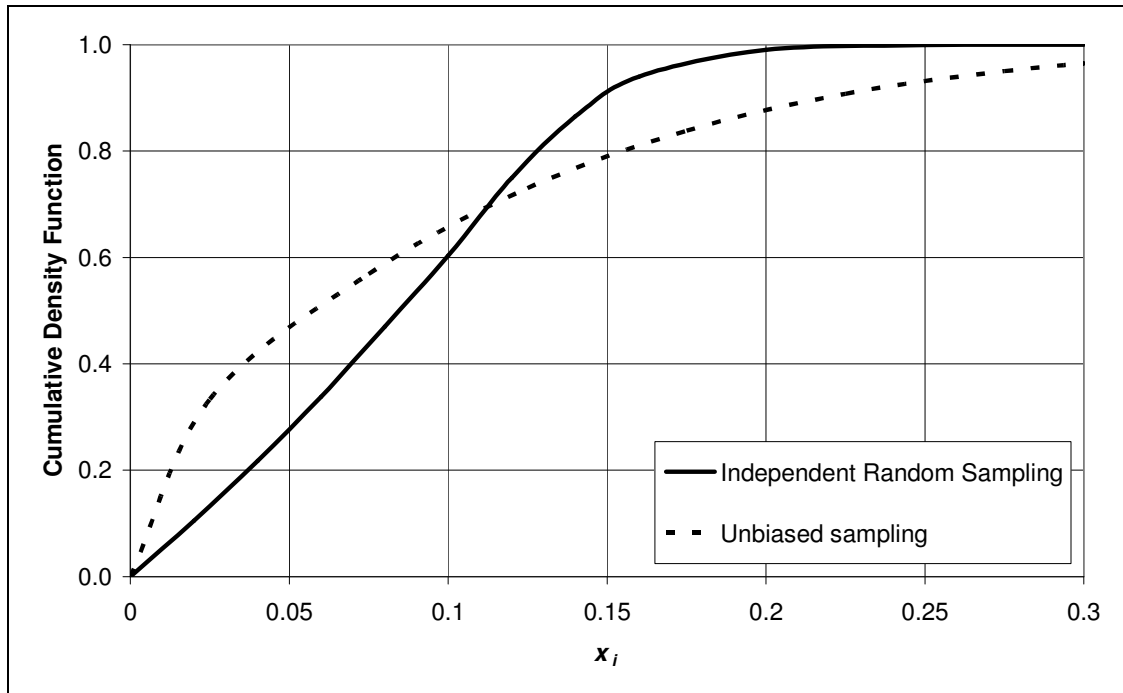
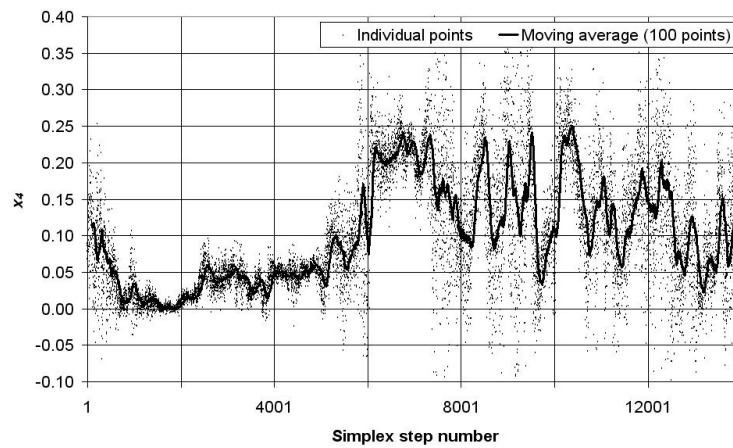
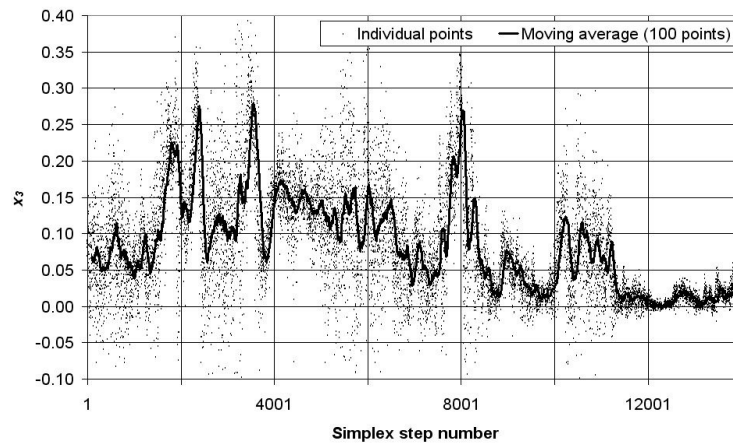
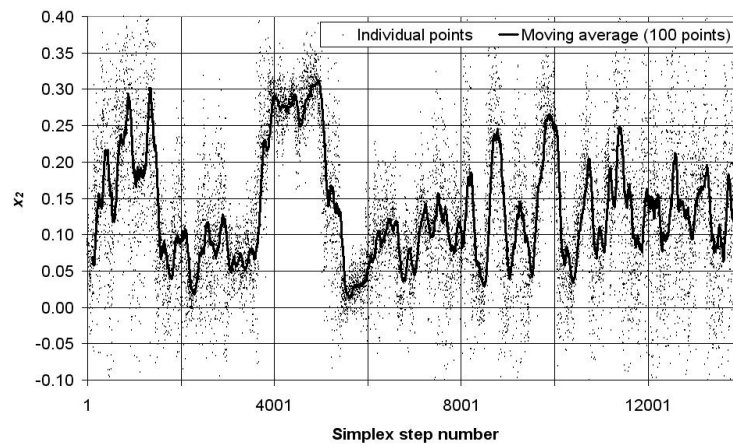
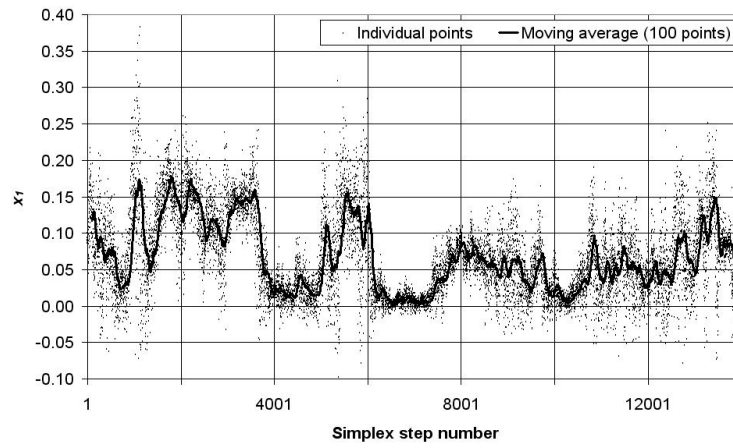


Figure 8-2: Sampling bias from inequality constraint boundaries $0 \leq x_i \leq 0.6$, for high temperature continuous variable SA, given the equality constraint: $x_1 + x_2 + x_3 + \dots + x_7 = 0.6$. Sampling bias in this example decreases the probability of searching adjacent to the inequality boundaries.

Figure 8-2 shows that inequality constraint does result in some sampling bias. For this particular example, bias resulted in a decreased probability of sampling adjacent to the boundary. Bias caused by the inequality constraint boundaries can be expected to reduce as the annealing temperature is decreased, due to the decreased probability of the simplex encountering these boundaries.

Figure 8-3 illustrates the behaviour of the individual decision variables for the high temperature SA in Figure 8-2. A particular feature from this figure is the auto-correlation between simplex steps (illustrated by the high variance in the moving average), an issue that is discussed further in Section 8.5. Another feature is that while searching is diverse (since with time all regions of the solution space are searched), a large number of steps are required to move from one region of the solution space to another. For example, for the decision variable x_7 , after 11,000 steps the simplex contracts in this dimension (adjacent to the boundary of $x_7 = 0$), and remains contracted (near this boundary) for about 3,000 steps. As the number of decision variables increases, the auto-correlation between steps increases, which means more time is required for diverse searching of the solution space. The large number of steps taken to transverse from one region of the solution space to another is a result of the solution space becoming very large when the number of effective decision variables is high, and hence the distances between two points (expressed as number of steps) can become very large.



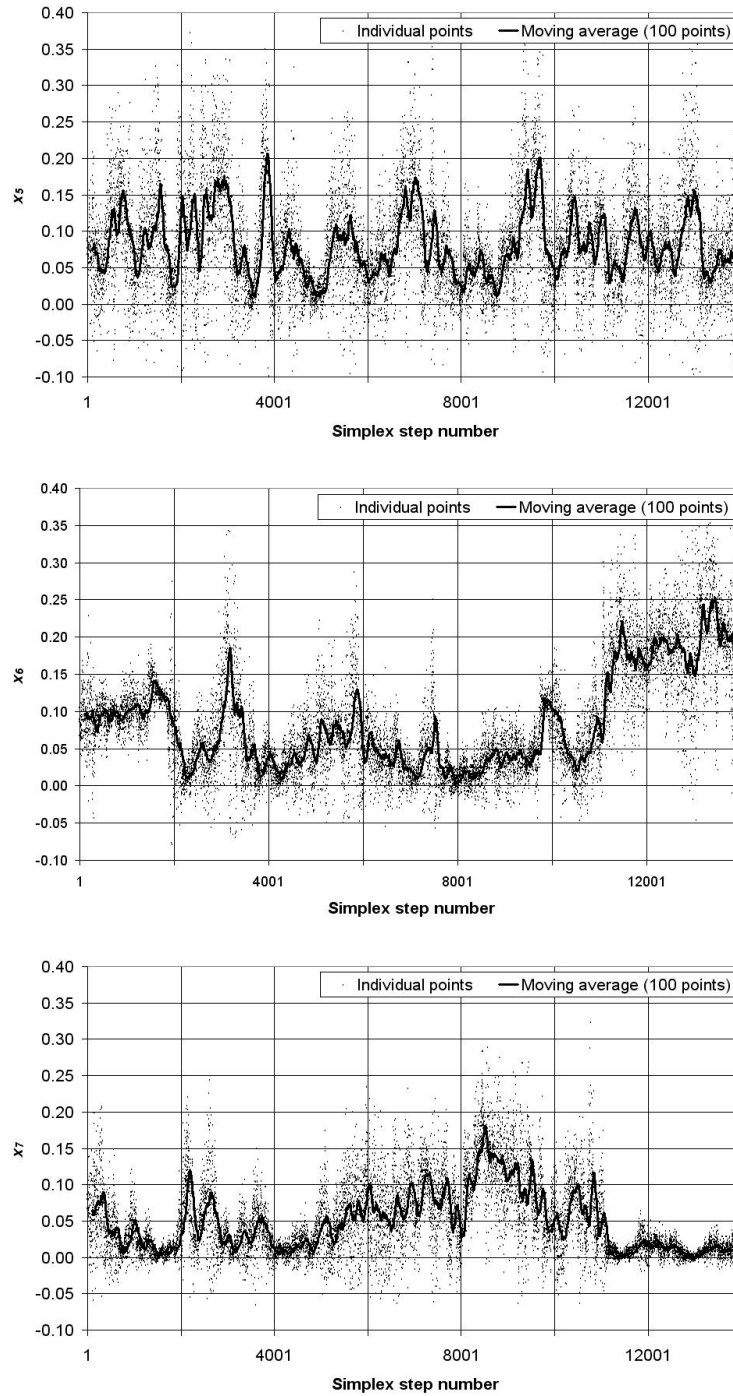


Figure 8-3: Analysis of individual decision variables using high temperature continuous variable SA, subject to constraints $0 \leq x_i \leq 0.6$ and $x_1 + x_2 + x_3 + \dots + x_7 = 0.6$, illustrating auto-correlation between simplex steps, and diverse searching of the solution space given a sufficient number of simplex steps

The effect of both equality and inequality constraints on geometric simplex behaviour is illustrated in Figure 8-4. For this example, the simplex is constrained within a triangular region by inequality constraints $0 \leq \tau[i] \leq 1.0$, and within a plane by the equality constraint $\tau[1] + \tau[2] + \tau[3] = 2.0$.

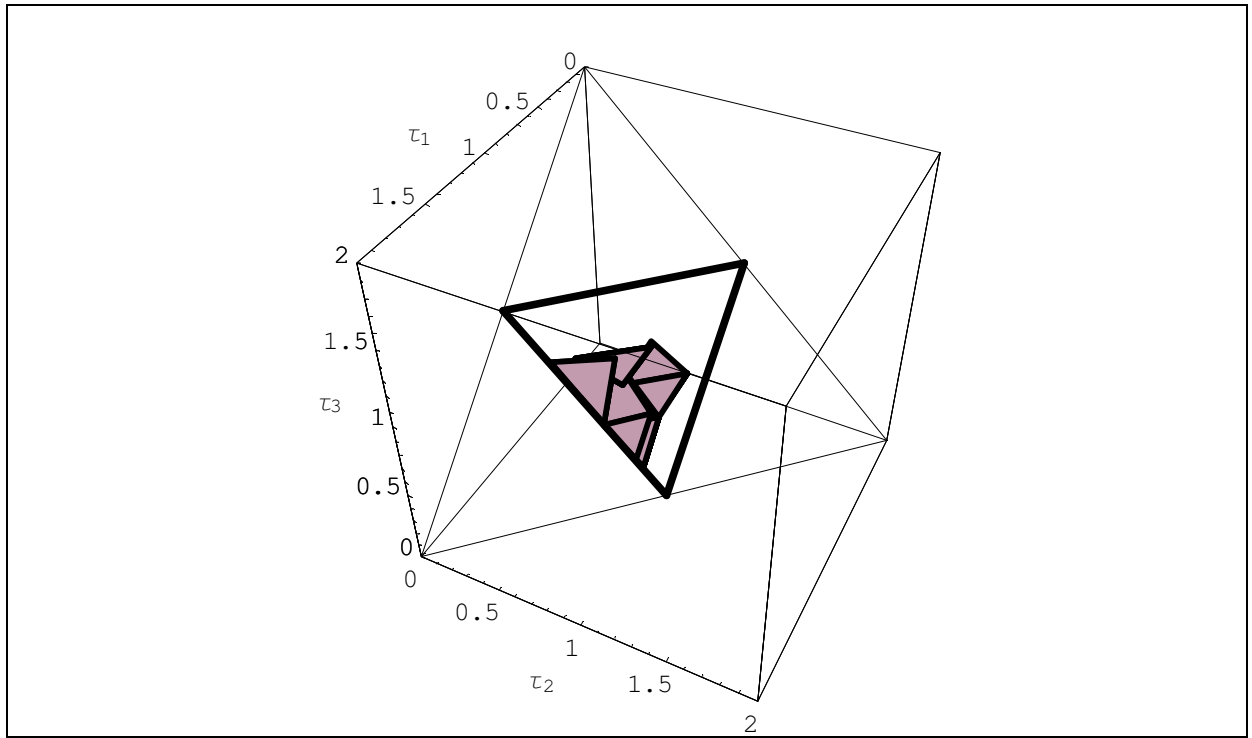


Figure 8-4: The geometric effect of an equality constraint (plane of $\tau[1] + \tau[2] + \tau[3] = 2.0$) and inequality constraints (bold line - $0 \leq \tau[i] \leq 1.0$) on Nelder Mead simplex behaviour.

For the inequality constraint, consideration was also given to alternative boundary profiles, other than a single very large penalty term applied as soon as any constraint violation occurred. In particular, a grace region was trialled, where the penalty term was not applied unless the constraint was exceeded by some defined threshold (e.g. for a grace region of 0.1, inequality constraints become $-0.1 \leq x \leq 1.1$ instead of $0 \leq x \leq 1.0$). It was reasoned that this would improve searching adjacent to the boundary, which may be of advantage given that there is an increased probability of superior solutions having at least some of the decision variables on the boundary (particularly for highly constrained problems). However, this alternative boundary profile gave inferior results compared with the simpler profile used. This poorer performance was in part due to difficulties caused by the interaction with the inequality constraints. Another variant trialled was to allow the simplex to ‘deform’ when it came in contact with the boundary, by moving back any vertex that violated the boundary so that it was on the boundary. This avoided the need to use a penalty term; however, difficulties arose as contact with the boundary sometimes resulted in the simplex becoming flatten against the boundary and unable to escape the particular boundary plane.

8.4 Simulated Annealing – Discrete Space

SA in discrete space was implemented to provide a comparison to continuous variable SA for two reasons: (a) it provided a method for incorporating constraints without the use of any penalty functions, and (b) it allowed an experimental comparison between optimising in discrete space and optimising in continuous space. The particular method of discrete-SA implemented was based on the

discrete SA method Press et al. (2002) used to solve the Travelling Salesman Problem. Decision variables were $\mathbf{A}_B[i,j]$ from Equation 6-1 ($\mathbf{A}_B[i,j]$ = water allocated to Block-TAP ‘buckets’). A local step was defined as follows:

- a) A contributing bucket is randomly selected from all buckets that are not empty.
- b) A receiving bucket is randomly selected from all buckets that are not full (i.e. do not violate Block (SC_B) or farm (SC_F) system capacity constraints in Equation 6-1). Furthermore, the receiving bucket cannot be the contributing bucket.
- c) Water (or allocation) is transferred from the contributing bucket to the receiving bucket. The amount transferred is the lesser of:
 - i) The amount available in the contributing bucket;
 - ii) The maximum amount the receiving bucket can receive without violating Block or farm system capacity constraints; or
 - iii) A specified maximum transfer amount.

After each local step, the new point was evaluated within the objective function, using Equation 6-2 to convert between allocation and a Farm Irrigation Strategy. If the performance of the new point was better than the previous point, the local step was always accepted. If it is worse, acceptance of the move was stochastic (refer Section 5.3.3). Unlike SA for continuous variables, the solution space was explored via a single trajectory (or ‘rover’), rather than via a simplex. One advantage of this method over continuous variable SA is that it is not necessary to use a penalty function to enforce boundary conditions, since every local step results in a solution that satisfies constraints. One disadvantage is that the irrigation options, which can be explored, are restricted due to the discretisation of the solution space. The degree of discretisation is controlled by the maximum transfer amount. If the maximum transfer amount has a small value, the negative effect of discretisation is reduced; however, the speed the rover moves through the solution space is also reduced, due to a smaller local step size. The rover speed was fixed, which is in contrast to continuous variable SA, where the simplex size (and therefore the simplex speed) automatically adjusts in response to the temperature and terrain. A variable rover speed would be relatively easy to implement by dynamically changing the maximum transfer amount, however this was not done due to other disadvantages of this method compared with continuous variable SA.

The general behaviour of the algorithm is illustrated in Figure 8-5. It used a FarmSim model (Good 2005) with a single soil layer and single crop coefficient. The farm consisted of three blocks, and the season was divided in nine Time Aggregation Periods, resulting in a total of 27 buckets (decision variables). The maximum transfer amount was 20 mm/ha¹⁸. The particular optimisation run was for

¹⁸ For this example, the objective function was a variant of Case Study 4, Chapter 9.

the first day of the irrigation season¹⁹. Results are presented as a percentage of the best known solution. The best known solution for this particular example was produced by continuous variable SA run (Table 8-1).

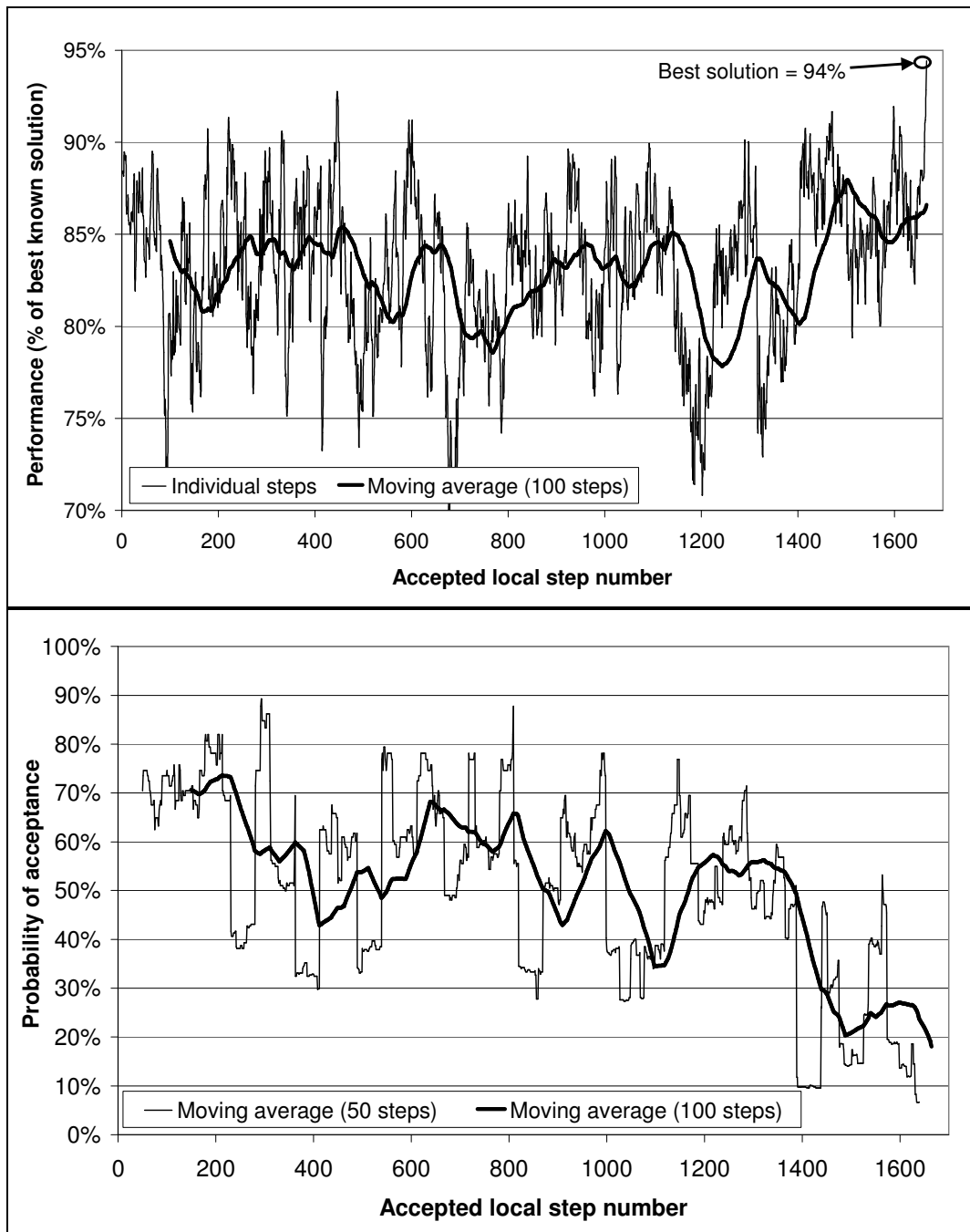


Figure 8-5: Discrete space simulated annealing – example annealing profile for 27 decision variables, illustrating in particular the decreasing probability of a local step being accepted with increasing performance. Performance is expressed as a % of the best known solution which was found using continuous variable SA (Table 8-1). Accepted local steps are evaluation points which become a vertex of the simplex for the next iteration.

¹⁹ When the SA Scheduler is used for scheduling irrigation, the optimisation procedure is run multiple times throughout the irrigation season (see Chapter 6). Only one of these optimisation runs is presented.

Overall, the algorithm performed poorly compared to continuous variable SA. At the beginning of annealing, when the temperature was high, the rover moves randomly within the solution space. During this phase, average performance is equal to the average of the solution space, and the probability of a step being accepted is on average 50%. As the temperature is reduced, the mean performance increases, while at the same time the probability of a step being accepted decreases. As the temperature approached zero, the probability of a point being accepted was below 10%. The 1650 accepted steps required about 4700 points to be evaluated. For this example, the best solution found by the optimiser fell well short (only 94%) of a solution obtained by continuous variable SA (Table 8-1). Furthermore, the method appears to offer no advantage over random searching of the solution space (Figure 8-6 and Figure 8-7). The primary reason for the poor performance is probably due to an issue highlighted by Press et al. (2002), that discrete variable SA is inefficient (where it is possible to pose the optimisation problem in continuous space) since when a point has above-average performance, though local downhill moves generally exist, it almost always proposes an uphill move (where optimum = minima). Using the example in Figure 8-5 (where optimum = maximum), even at a moderate performance of around 94% of the best known solution, only 5% of steps trialled resulted in an improvement – an uphill gradient. Even the 5% of the time an uphill gradient was found, it is unlikely this gradient would be in the direction of steepest ascent. Furthermore, the greater a solution is above average solutions, the lower the probability of a step being in the direction of an uphill gradient.

8.5 Random Sampling of the Solution Space

Continuous variable SA requires the setting of two main parameters: (a) the initial temperature, and (b) the rate of cooling. Setting of the latter parameter is discussed in Sections 8.7. Setting the initial temperature requires knowledge of the variability of the solution space. If the initial temperature is too low, the simplex will be unable to escape localised depressions (where optimum = minima), resulting in only a portion of solution space being accessible for exploration. If the initial temperature is too high, excessive time is wasted searching in an almost totally random manner. Since the variability of the solution space will vary depending on the particular characteristics of the farm being modelled, the approach taken was to directly measure variability via random sampling of the solution space prior to SA, and to set the SA initial temperature proportional to the variability.

Using the SA analogy to metallic annealing, the concept of a ‘melting point’ was used to express variability. Physically, the melting point of a material is defined as the temperature at which atomic movement at a local scale is essentially random. For the SA equivalent, the melting temperature was defined as two times the standard deviation of a random sample of the solution space. At this temperature, simplex movements are essentially random.

Three methods were trialled for randomly sampling the solution space: (a) high temperature continuous variable SA, (b) high temperature discrete variable SA, and (c) Independent Random

Sampling. Both SA methods were run at temperatures significantly greater than the melting temperature, such that all step/simplex movements were random. Equation 8-1 presents the Independent Random Sampling algorithm. The behaviour of the two SA methods is illustrated in Figure 8-6 and Figure 8-7. These examples use the same objective function as Section 8.4; therefore, for the discrete variable SA, this corresponds to 27 decision variables, and for continuous variable SA, this corresponds to 26 effective decision variables (the difference is due to continuous variable SA ability to use equality constraints to reduce dimensionality). A moving average of 250 consecutive steps is plotted in these figures to highlight auto-correlation.

FOR EACH point:

- Randomly generate values for decision variables $\mathbf{B}[i,j]$ & $\tau[i]$ (see Equation 6-1) between the upper and lower limit of each variable
- Scale $\mathbf{B}[i,j]$ & $\tau[i]$ such that equality constraints Equation 6-1 [f] & [g] are satisfied

$$\tau = \tau * \left(\frac{SC_F}{SC_B * \text{Sum}(\tau)} \right)$$

FOR $i=1$ to N_T

$$\mathbf{B}[\text{row } i] = \mathbf{B}[\text{row } i] * \left(\frac{SC_F}{SC_B * \text{Sum}(\mathbf{B}[\text{row } i])} \right)$$

- IF $\mathbf{B}[i,j]$ & $\tau[i]$ are still between the relevant limits

Use point

- ELSE

Discard point

Equation 8-1: Method of Independent Random Sampling, for generating random and independent solutions to Equation 6-1.

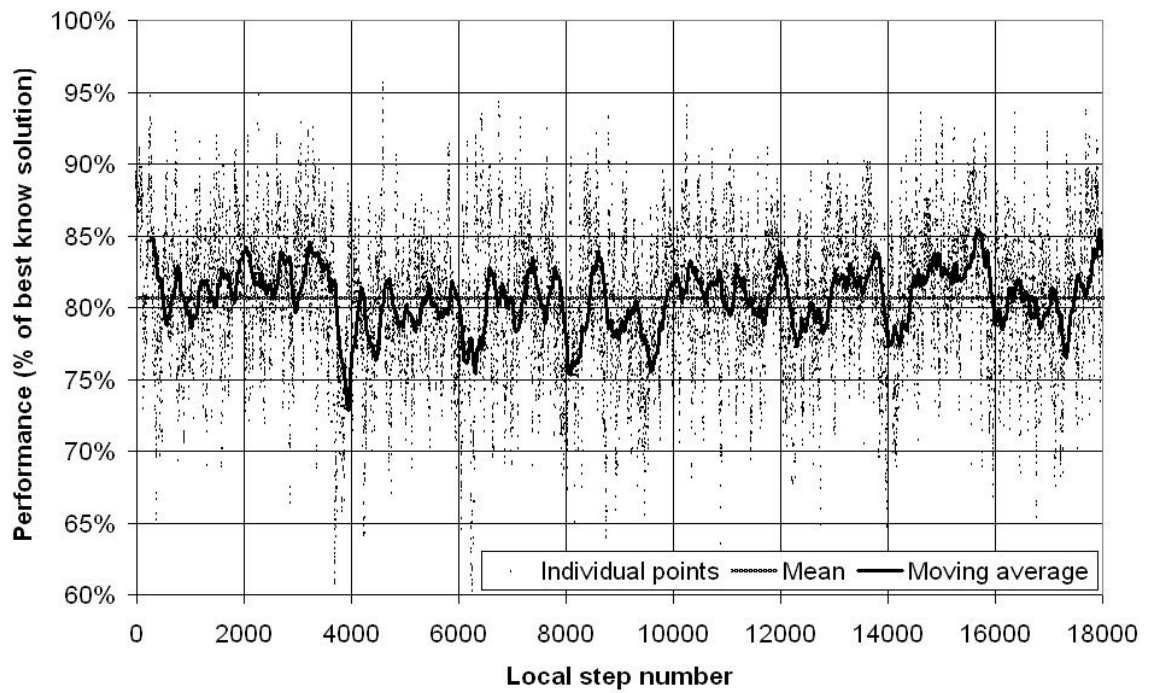


Figure 8-6: Random sampling using high temperature discrete variable SA, illustrating high auto-correlation. Performance is expressed as a % of the best known solution which was found using continuous variable SA (Table 8-1).

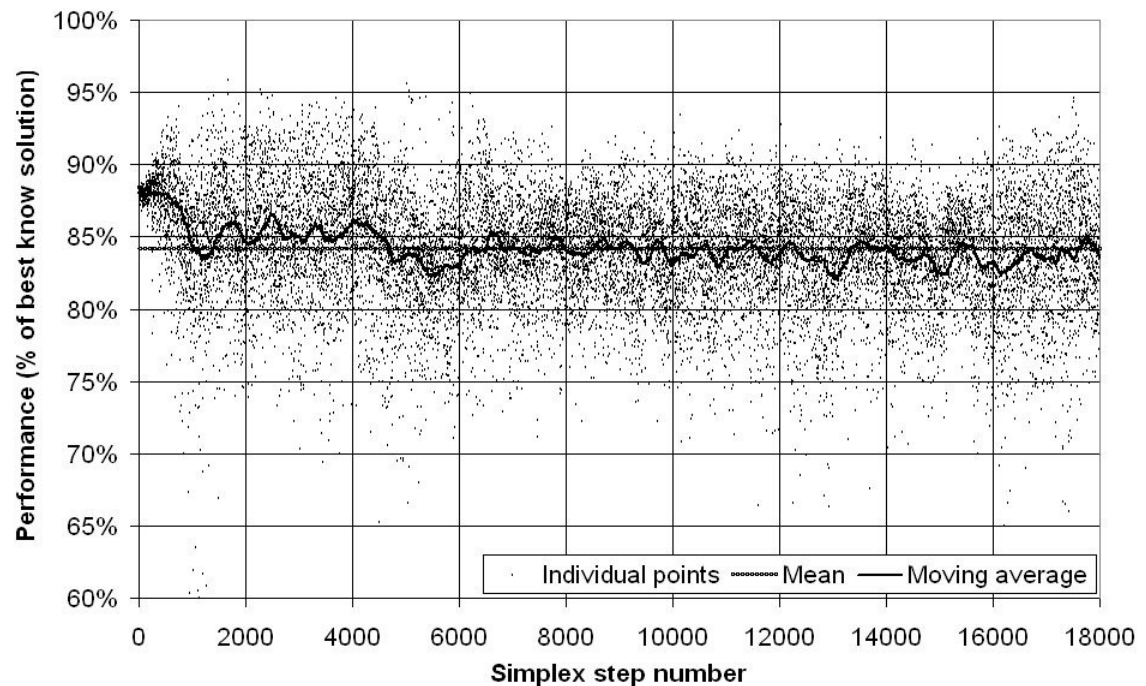


Figure 8-7: Random sampling using high temperature continuous variable SA, illustrating some auto-correlation. Performance is expressed as a % of the best known solution which was found using continuous variable SA (Table 8-1).

From Figure 8-6 and Figure 8-7, the difference in the mean of the two methods may be attributed to the sampling bias within continuous variable SA, caused by the inequality constraints (see Section 8.3.2). The standard deviation of the individual points was 0.048 and 0.038 for Figure 8-6 and Figure 8-7, respectively, while the standard deviation of the 250 point moving average was 0.020 and 0.011, respectively. Two points of interest from these figures are: (a) the high auto-correlation, and (b) the ability for random searching to match the performance of the discrete variable SA from Figure 8-5²⁰. Auto-correlation between consecutive steps is demonstrated through the higher than expected variability of the moving average. Were consecutive points evaluated independently, the expected standard deviation of the moving average would be 0.0031 and 0.0024 for Figure 8-6 and Figure 8-7, respectively²¹ – an order of magnitude less than what was observed. Auto-correlation was greater for discrete variable SA than for continuous variable SA. The second point of interest is the ability for random searching to match the performance of the discrete variable SA. In Figure 8-6 and Figure 8-7, a total of 8 and 32 solutions (0.4/1000 and 1.8/1000 objective function evaluations), respectively, outperformed the best solution found in Figure 8-5. Meanwhile, Figure 8-5's best solution required a total of 4700 (0.2/1000) objective function evaluations. This suggests that for this particular optimisation problem, discrete variable SA may offer no advantage over randomly searching the solution space.

The main advantage of the Independent Random Sampling method given in Equation 8-1 is that because there is no auto-correlation between sample points. However there is the sampling bias caused by the equality and inequality constraints. Figure 8-8 illustrates this bias, by comparing the CDF of decision variables x_i (where x is a surrogate for Equation 6-1 decision variables $\tau[i]$ or $\mathbf{B}[i,j]$) for the Independent Random Sampling method with unbiased sampling. The CDF for unbiased sampling was obtained by discretising x_i and generating all possible solutions. For this example, the equality constraint is $x_1 + x_2 + x_3 + \dots + x_7 = 0.6$ and the inequality constraints are $0 \leq x_i \leq 0.6$.

²⁰ Where the temperature eventually reduces to zero unlike high temperature SA in Figure 8-6

²¹ $0.048/\sqrt{250}$ and $0.038/\sqrt{250}$

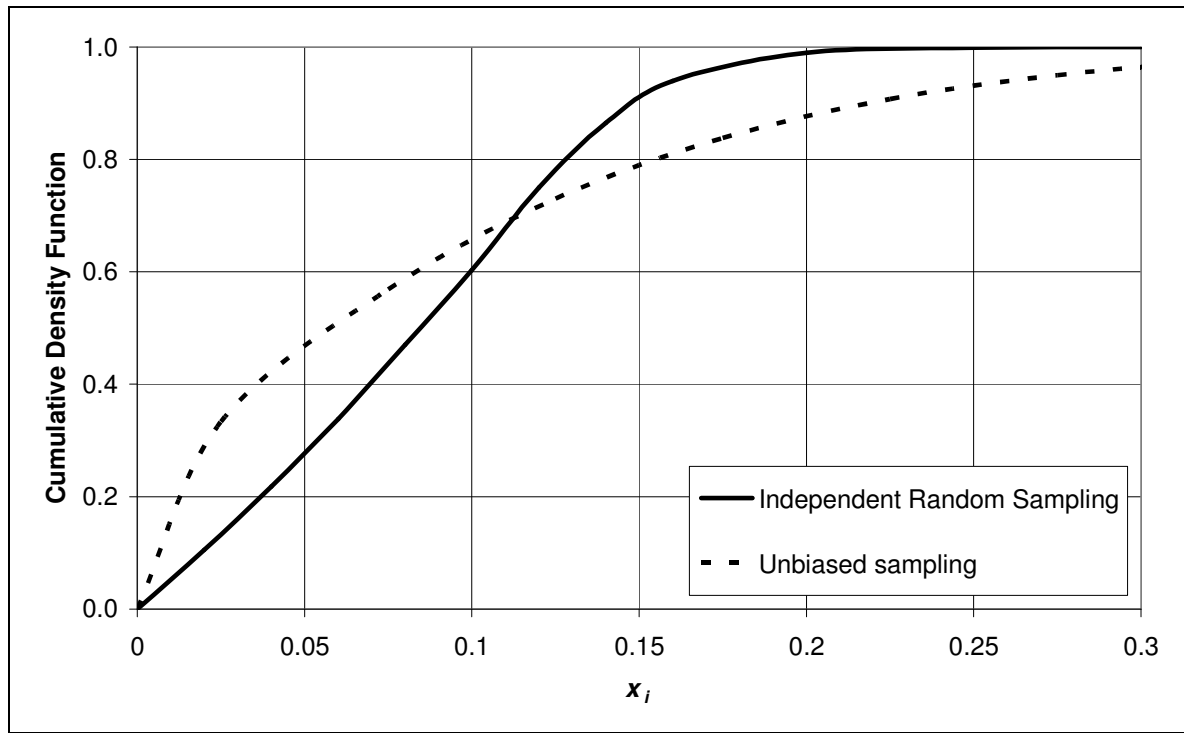


Figure 8-8: Illustrating sampling bias for Independent Random Sampling, given constraints $x_1 + x_2 + x_3 + \dots + x_7 = 0.6$ and $0 \leq x_i \leq 0.6$. Sampling bias in this example decreases the probability of searching adjacent to the inequality boundaries.

Because Independent Random Sampling avoided the problem of auto-correlation this was used as the method for random sampling within the SA scheduler in Chapter 9 and 10.

8.6 Starting Simplex

Two modifications were made to Press et al.'s (2002) continuous variable SA method (as implemented by CenterSpace Software (2004)) with regards to the starting simplex. The first change was to allow the size of the initial simplex to be defined. The second change was to specify a simplex that satisfied all equality constraints, and had the number of vertices reduced by the number of equality equations. Details of this algorithm can be found in the electronic copy of the computer code appended.

The starting point(s) were taken as either the point where the trigger soil moisture level was constant for all TAP and Blocks, or, in the case of Population Analysis (Section 8.8), points obtained from random sampling.

8.7 Convergence

The continuous variable SA optimiser was run multiple times in order to gain an understanding of how annealing behaviour was affected by the initial temperature and the rate of cooling. Two sets of results are presented. For the first set (Figure 8-9 and Table 8-1), the objective function was as per

Case Study 3, Chapter 9. It used a FarmSim model (Good 2005) with a single soil layer and single crop coefficient. The farm consisted of two blocks, and the season was divided in ten TAP, resulting in a total of 19 effective decision variables. The particular optimisation run was for the first day of the irrigation season.

Figure 8-9 illustrates how annealing profiles varied in response to differences in initial temperatures. For clarity, the profiles were plotted on two graphs. The temperature is reduced in a step-wise manner (referred to as Temp. step 1, 2...). The first temperature step is the initial temperature. The duration of each step is equal to the number of simplex steps per temperature step. At the end of each step the temperature is decreased by a constant amount, such that for the final step the temperature is zero.

Table 8-1 illustrates the effect of the initial temperature, the rate of cooling, and the variability in solutions between identical runs. The rate of cooling is defined by two parameters: the number of temperature steps and the number of simplex steps per temperature step. Duplicate runs are indicated by a decimal run number (e.g. 7.0, 7.1, and 7.3). Two values of performance are presented: the value the optimiser converges to when the temperature is reduced to zero, and the best value found anytime during the annealing process.

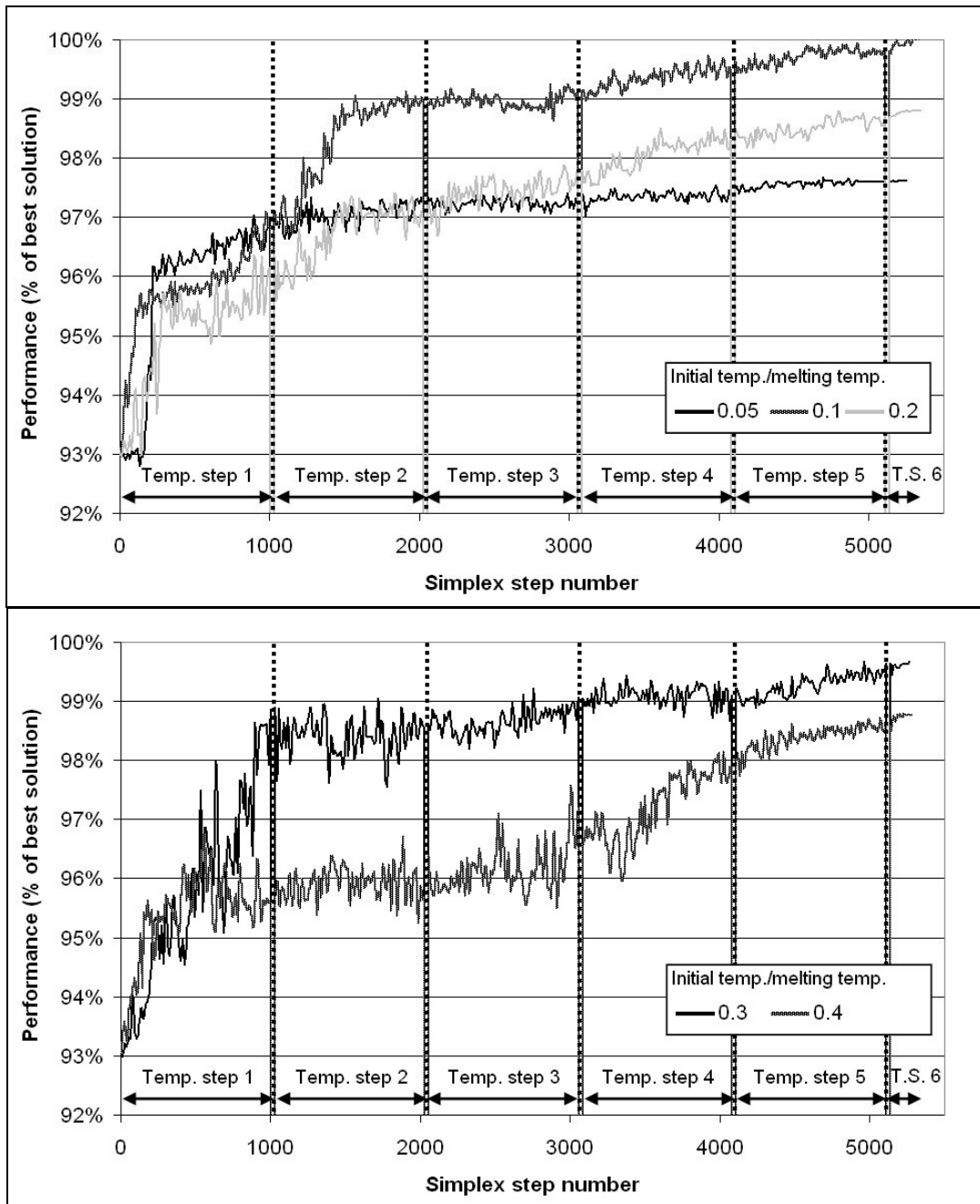


Figure 8-9: Continuous variable SA profiles with 19 effective decision variables illustrating convergence to regional rather than global optimum and the influence of initial temperature. Performance is expressed as a % of the best known solution, which was found using continuous variable SA (Table 8-1).

Table 8-1: Continuous variable SA – illustrating the influence of initial temperature and cooling rate on performance.

Run No.	Initial temp. / melting temp.	No. temp. steps	Simplex steps per temp. step	Convergence value (% of best solution)	Maximum value (% of best solution)
1	0.20	6	500	98.2%	98.2%
2	0.20	6	1000	98.8%	98.8%
3	0.20	6	2000	99.1%	99.2%
4.0	0.30	6	1000	99.7%	99.7%
4.1	0.30	6	1000	98.5%	98.5%
4.2	0.30	6	1000	99.0%	99.1%
5	0.40	6	1000	98.8%	98.8%
6	0.30	6	2000	98.9%	98.9%
7.0	0.10	6	1000	100.0%	100.0% ⁽¹⁾
7.1	0.10	6	1000	99.2%	99.2%
7.2	0.10	6	1000	98.1%	98.1%
8	0.05	6	1000	97.6%	97.6%
9	0.10	6	2000	98.2%	98.2%
10.0	0.30	6	2000	99.0%	99.0%
10.1	0.30	6	2000	97.5%	97.5%
11	0.30	4	500	97.0%	97.0%
12	0.30	4	1000	96.9%	97.1%
13	0.30	8	500	97.8%	97.8%
(1) Best known solution					

SA performance was stochastic, varying even between runs that had the same annealing parameters (but different random number seeds). This stochastic behaviour made it difficult to define optimum annealing parameters, since particular annealing parameters that performed well on a given run, may perform poorly in a subsequent run. However, some general conclusions could be drawn.

The main feature of Figure 8-9 is that the optimiser is converging toward some optimum. For these examples, rapid improvements are made initially (within the first temperature step), generally followed by a period of slow but steady improvement. Generally, the best solution found throughout the annealing procedure occurred when the temperature had been reduced to zero – an observation that supports the statement that convergence is occurring. However, since each SA run does not converge to the same solution, it is known that convergence cannot be to the global optimum. Rather, it is to some regional optimum. For certain runs (e.g. initial temp./melting temp.=0.05) the performance plateaus and remains near constant for a large number of simplex steps, suggesting that an adaptive cooling regime may be of benefit.

In Table 8-1, the best solution occurred in run number 7.0. Generally, the final convergence and minimum values are the same. As mentioned above, this supports the statement that convergence is

occurring. The main conclusion from this table is that both the initial temperature and the cooling rate had an influence on the performance of the continuous variable SA; however, defining optimal parameters without a large number of duplicate runs is difficult due to the variability in performance between identical runs.

The second set of the results is presented in Figure 8-10, which illustrates convergence to numerous regional minima after starting from randomly selected starting points. For these runs, the objective function was the same as in Section 8.4. For continuous variable SA, this corresponds to 26 effective decision variables. Initial above average solutions were selected from randomly generating independent starting points and selecting the 100 best solutions from this sample. Each of these good solutions provided the starting point for a particular continuous variables SA run. SA annealing schedulers had an initial temperature of 0.3 times the melting temperature, and step-wise cooling as described above, with six temperature steps and 1000 simplex steps per temperature step. Figure 8-11 presents the correlation between initial starting point performance and the improved performance following SA.

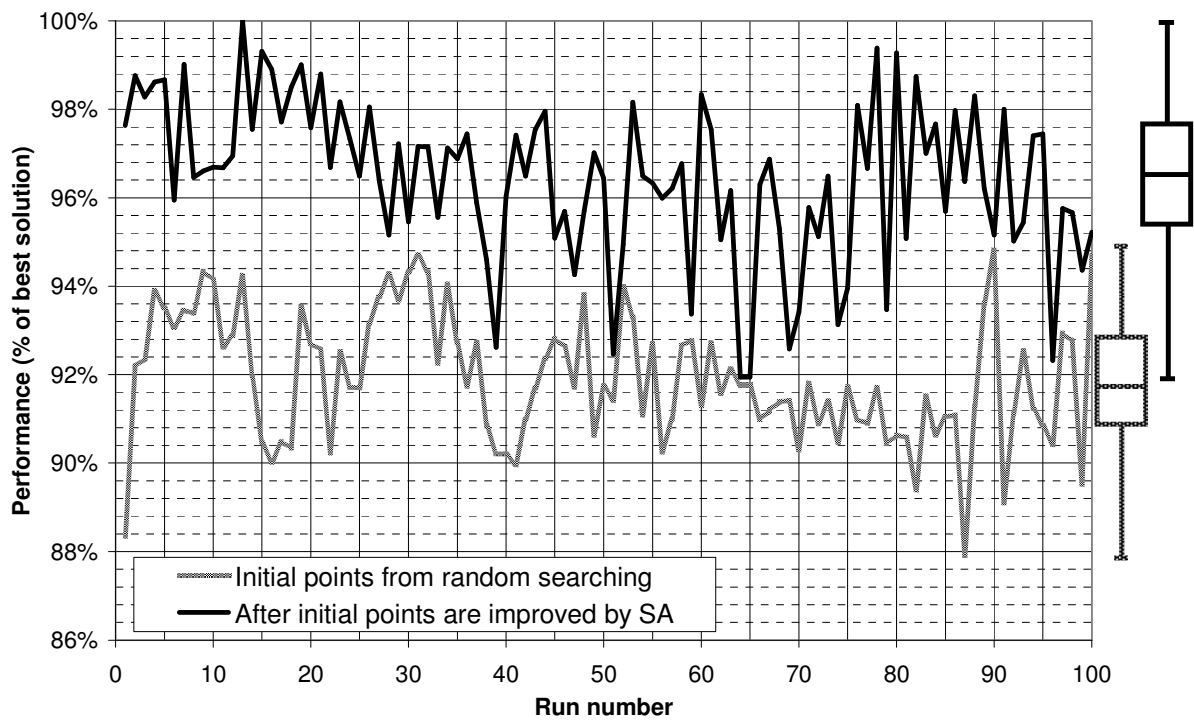


Figure 8-10: Continuous variable SA converging to numerous regional minima after starting from randomly selected (above average) starting points (given 26 effective decision variables).

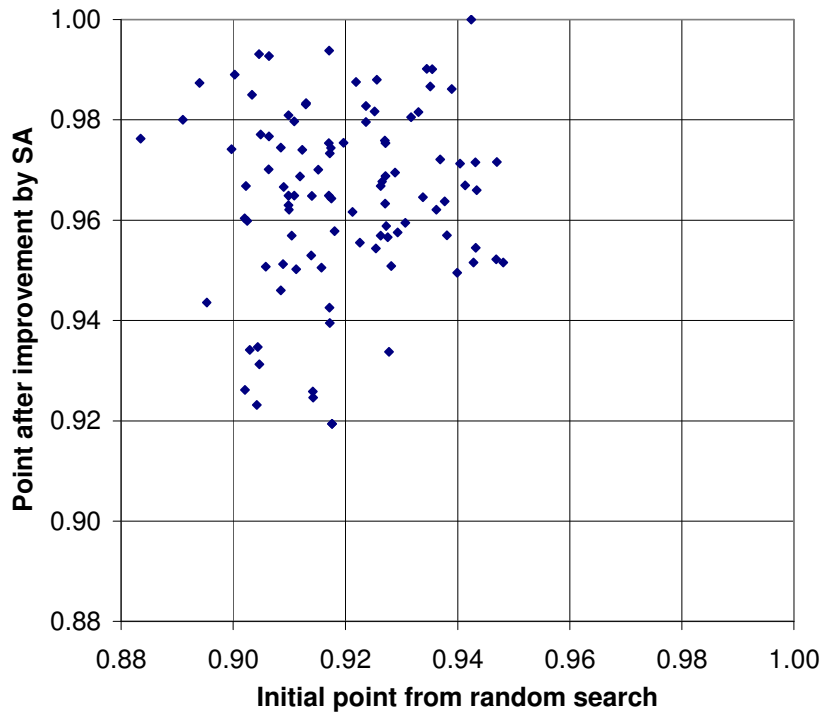


Figure 8-11: Correlation for Figure 8-10; between initial starting point performance and the improved performance following SA.

Figure 8-11 shows that there was no correlation between the performance of the initial starting point and the performance following improvement via SA.

It can be concluded from the above results that global convergence using single start SA was generally not possible since beyond a certain point, there was no significant benefit in longer annealing times and higher initial temperatures. Instead, the optimiser found regional rather than global optimum. The choice of which particular regional optimum found was stochastic. Higher initial temperatures, together with long annealing times, generally resulted in greater variability in performance, since the higher initial temperature allowed the simplex to migrate to parts of the solution space other than the region the simplex was started in.

The above observations can be explained by the theoretical behaviour of SA. SA with continuous variables can be considered an extension of local minimisation gradient methods. However, unlike steepest descent methods, not just local minima but regional depressions can be identified. A regional depression is a region of the solution space, which *on average* has lower values than the surrounding solution space. The size of the region that averaging occurs is a function of the annealing temperature. Within the solution space, there may be several (or even multitudes of) depressions that have a scale equal to the averaging area. The larger and deeper the depression, the greater the probability the simplex will be found in that depression. As the temperature is reduced, the simplex will be confined to search within a single depression. The choice of this depression is stochastic; the probability is higher for larger depressions. This process of confinement within a particular depression continues at

increasingly smaller scales until local minimisation occurs. Alternatively expressed, at high temperatures the search focuses on finding large depressions, while skipping over small depressions. As the annealing temperature cools, the search is confined to a single large depression and increased attention is given to exploring smaller depressions within the particular large depression. The consequence of this behaviour is that at each scale of averaging; only one of many depressions is explored at a more detailed level.

A single start SA is therefore only able to find one of many regional optimums. It is therefore possible that many other good, or even far superior, regional optimums may exist. This problem was addressed in two ways. The first simple method was to run the optimiser multiple times starting from different solutions found from Independent Random Sampling (Equation 8-1), thereby allowing more than one regional optimum to be found and the best of these selected. The greater the number of regional optimum found, the better the expected overall minima. The second method extends this first method by utilising common characteristics between superior solutions (e.g. regional optimum), and is described in Section 8.8.

8.8 Population Analysis

The problem with running continuous variable SA only once is that only one of many regional optimum are explored. Section 8.7 suggested that one method to mitigate this problem was to run SA multiple times (starting from different random points using Independent Random Sampling [Equation 8-1], thereby allowing more than one regional optimum to be explored) and then select the best of these solutions.

This section extends the method of multiple annealing runs by identifying common characteristics between several similar superior solutions (e.g. regional optimum), and using this information to isolate and explore the depression, which all these superior solutions fall within. Utilising population information is an important aspect of certain heuristic methods – in particular, Genetic Algorithms. The method of Population Analysis proposed in this section provides one mechanism of incorporating beneficial information contained within populations (but not available from considering individual solutions independently) into the constrained continuous variables SA method described in Sections 8.2 and 8.3.

Subsequent discussion will refer to different scales of depressions (e.g. local, regional, macro-regional), which are illustrated in Figure 8-12. This figure is purely illustrative. In particular, while in practice the solution space will be multi-dimensional, this figure is two dimensional. Two-dimensional space with a complex terrain (with macro-regional depressions, containing many regional depressions, which themselves contain a multitude of local minima, as in Figure 8-12) may be visualised as analogous to a higher dimensionality solution space. Meanwhile, two-dimensional space

with only a few local maxima and minima may be visualised as analogous to a low dimensionality solution space.

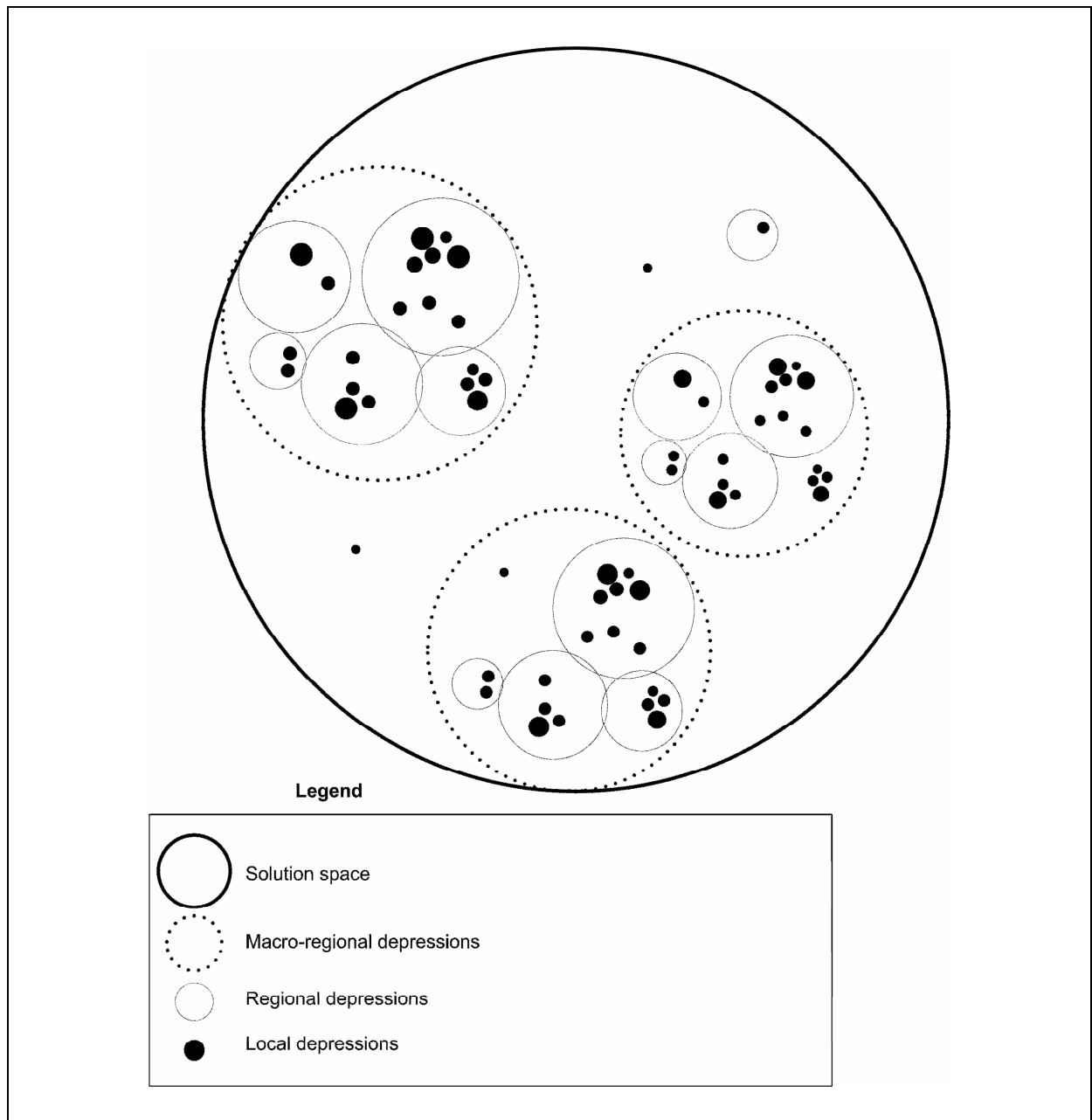


Figure 8-12: Various scales of depressions (where optimum = minima).

The proposed method uses population information to **identify**, **isolate** and **explore** depressions at increasingly smaller scales. These phases are described below. Further details may be found in the electronic copy of the source code appended.

The first phase is identification. For the example given in Figure 8-12, the first depression scale would require identification of macro-regional depressions. The particular method of identification used involved a four-step process:

- a) The first step used Independent Random Sampling (Equation 8-1) to generate a large sample of independent random points. From this sample, the best points (elite points) were selected.
- b) The second step involved improving these elite points via continuous variable SA.
- c) The third step involved mapping each of the elite points to binary form.
- d) The fourth step involved comparing the elite points (in binary form) to identify points that were similar; these points were grouped together. Macro-regional depressions were defined as the regions that encompass each of the groups of similar (superior) points.

The second phase is isolation of a particular macro-depression. An isolated region of a solution space has two main aspects: (a) reduced decision variables ranges, and (b) reduced dimensionality. Where the similar superior points (in binary form) have a particular bit in common, this bit is fixed provided all higher-order bits for that particular decision variable can be fixed. Fixing bits reduces a decision variables range. For instance, if only the first bit is fixed, the range would be reduced from $0 \leq x \leq 1$ to $0.5 \leq x \leq 1$ or $0 \leq x \leq 0.5$, depending on the value of the bit. If two bits are fixed, possible ranges are $0 \leq x \leq 0.25$, $0.25 \leq x \leq 0.5$, $0.5 \leq x \leq 0.75$ or $0.75 \leq x \leq 1.0$. For each additional bit that is fixed, the range of a decision variable is further halved. Where all bits for a decision variable are fixed, the entire decision variable is fixed. When the entire decision variable may be fixed, the number of effective decision variables and vertices of the simplex can be reduced by one.

The final phase is exploration of each of the macro-regional depressions that have been identified and isolated. The particular method used to explore these macro-regional depressions was single-start continuous variable SA. Alternatively (if macro-regions still contained a large range of decision variables and/or number of dimensions), regional depressions could be identified within the isolated macro-regions, by repeating the process of identification described above, but at a reduced scale (thus the exploration phase at one scale becomes the identification phase at a lower scale). Thus the method can be described as a Branch and Bound procedure which isolates and explores successively smaller regions.

The choice of the size of the random sample is related to the benefits of a larger sample traded against the computational cost of evaluating this sample in the objective function, and the requirement for a sufficient sample size to measure the melting temperature (see Section 8.5). A larger random sample has the advantage of being able to find better starting points. However, it is unclear whether or not better starting points result in better regional optimum (following improvement via SA), given the results from Figure 8-11, which showed no correlation between the starting point performance and performance following improvement via SA. Given the uncertainty of the benefits, 10% of the total computational time was considered an appropriate allocation of computational resources to random sampling.

The population size ideally would be a function of the number of dimensions and computational constraints. A population size of only four was used in the case studies in Chapter 9 that used Population Analysis. This is much smaller than is typical of Genetic Algorithms, but was required for computational feasibility. From this small population, the three superior solutions that were most in common were selected as the single and only group of ‘similar superior points’. Four was considered to be the minimum possible population size that can be used with Population Analysis, since a smaller population will result in a high probability of similar characteristics occurring by chance.

Standard binary coding allows identification of which decision variables have a similar magnitude, but not an identical value. For example, the first binary digit indicates whether the value is between 0-0.5 or 0.5-1.0. If similar solutions have a value for that decision variable of between 0-0.5, the first bit can be fixed. Fixing of subsequent bits is possible only if all preceding bits are fixed. Grey-binary coding was also considered but was not used, since without a Genetic Algorithm mutation-type operator, this method offers no advantage over standard binary coding. The number of bits used when converting from continuous to discrete space affects the likelihood of an entire decision variable being fixed. A decision was made to use six bits, since this meant that a decision variable would only be fixed provided the particular decision variable varied by less than 3% between similar superior points. The fixing of decision variables was only partially implemented due to research time constraints and was not used within the case studies in Chapter 9.

A sample of 20 optimisation runs was taken from the optimisation runs carried out in Chapter 9, for runs that had 2, 3, 4, 5 and 6 effective decision variables. The objective was to determine whether there was any benefit in using Population Analysis compared with simply running SA multiple times. The results are presented in Table 8-2.

Table 8-2: Benefits of using Population Analysis compared with only using multi-start continuous variable SA.

No. effective decision variables	Average % of bits fixed	% of time isolation improved performance' ⁽¹⁾
6	32%	55%
5	32%	70%
4	49%	65%
3	70%	85%
2	93%	100%
(1) Without any isolation, 25% of the times an additional SA run can be expected to result in a point better than the three ‘similar superior points’.		

Table 8-2 strongly suggests that isolation of a macro-depression using Population Analysis did result in more efficient optimisation. If there were no benefits in Population Analysis, running SA after a macro-regional depression had been isolated would have resulted in an improved solution (compared with the three similar superior points) occurring only 25% of the time. However, after isolation, most

times the SA was rerun resulted in an improved solution, indicating the probability of finding a very good solution is higher within a macro-depression than other parts of the solution space. The benefits appeared to increase with a decreasing number of effective decision variables.

Further investigation into the potential benefits and limitation of Population Analysis are required. One particular aspect not explored was the benefits of fixing of decision variables. However, initial positive results would suggest the basic concept will generally improve optimisation efficiency, particularly at higher dimensionality when there are several macro-regional depressions and a multitude of regional depressions.

8.9 Alternative Options Considered

The use of other heuristics was considered. As discussed previously in Chapters 5 and 6, standard Genetic Algorithms would not appear a promising method for optimisation within the irrigation scheduler, due to the scheduler's high dependence between decision variables, and the need to optimise in discrete rather than continuous space. Furthermore, the recombination operator is not well suited for incorporation of the equality constraints. Consideration has been given to a Genetic Algorithm variant termed Population Reinforced Optimisation Based Exploration (PROBE) (Barake et al. 2003), since this algorithm was designed to incorporate boundary conditions, and has subsequently been shown to perform well relative to other heuristics, when applied to other problems. However the approach proposed in this chapter will make better use of gradient information and therefore is likely to be superior. Furthermore, the use of multiple generations required for PROBE is likely to be computationally infeasible when using complex objective function farm simulations such as the FarmWi\$e model used in Chapter 10.

Previous authors had found that Tabu search can perform poorly when optimising at higher dimensions (Chapter 6). This method was not implemented since it was not expected to offer any benefits over continuous variable SA.

8.10 Conclusions

The comparison between continuous and discrete SA showed that optimising in continuous space was much more efficient than optimising in discrete space. Two main extensions to continuous variable SA were the incorporation of equality constraints and utilisation of population information. The incorporating of the equality constraints from Equation 6-1 into continuous variable SA improved optimisation efficiency over alternative methods of incorporating this constraint. The novel method termed Population Analysis, which utilised population information, showed that such information could be used to increase optimisation efficiency. Ideas from this chapter, in particular incorporating linear equality constraints and population information into continuous variable SA, may be useful in other optimisation problems that have a similar structure.

9 CASE STUDY COMPARISONS WITH STOCHASTIC DYNAMIC PROGRAMMING

9.1 Context and Overview

This chapter compares the performance of the SA scheduler (Chapter 6) with irrigation scheduling using SDP. SDP can guarantee optimal scheduling (when performance is averaged over several seasons) if the objective function and climate assumptions within SDP are valid. This was done by making the case study simulation model identical to the system model embedded within SDP. While it is known that the model within SDP is too simple to be useful for scheduling irrigation in practice (Chapter 4), by making these simplifications the SA scheduler performance could be compared with the known optimum performance, thereby allowing partial testing of the following hypotheses:

- 1) That the method of automatic adjustment (within objective function simulations) of irrigation strategies, in response to deviations between expected and simulated water use (described in Section 6.5), is a reasonable approximation of the optimal adjustment strategy;
- 2) That the restricted range of irrigation schedules (as a consequence of decision variables describing general management decisions) still allows for close to optimal scheduling (Section 6.4); and
- 3) That the custom heuristic method (Chapter 8) can produce near optimum solutions, given sufficient calculation time.

The SA scheduler required assumption 1 due to the stochastic climate. Chapter 6 explains how decision variables dictate the priority that should be given to various crops at different times. The automatic adjustments of irrigation strategies (defined via trigger soil moisture levels) aimed to preserve the *relative* priority of each block. For the optimal management strategy, the hypothesis was that the relative priorities still remained optimal despite minor changes to trigger soil moisture levels. The second assumption is associated with decision variables describing general management strategies rather than individual irrigation decisions. The consequence of this assumption is that only a restricted range of irrigation schedules (which can be described in terms of these strategies) are able to be explored. The hypothesis was that the optimal irrigation schedule would still be within the domain of explorable options. In contrast, SDP does not require either of the first two assumptions.

The final assumption relates to the performance of the heuristic method developed in Chapter 8. While it is known that the algorithm cannot guarantee the global optimum, the method can find various regional optimums. SDP allows a comparison of how close the performance of the best of these regional optimums is compared with the global optimum found by SDP.

Previous multi-crop SDP irrigation scheduling algorithms were restricted by the computational power of computers at the time they were developed; therefore, some simplifications were required in order to reduce the computational demand. Section 9.2 makes use of the large computational power now available to extend a previously developed single-crop irrigation scheduling algorithm (that uses standard SDP) to multiple crops, thereby allowing some limitations of existing multi-crop SDP schedulers to be overcome.

Case study parameters are presented in Section 9.3, while results are presented in Section 9.4. The conclusion from these results was that the SA scheduler was able to closely match the performance of SDP, suggesting that the three assumptions listed above are reasonable.

9.2 Proposed SDP Irrigation Scheduling Method

9.2.1 Existing SDP algorithms

A number of authors have used SDP for scheduling irrigation for a single crop. Few significant improvements have been made to these single crop SDP schedulers since the work of Córdova and Bras (1979). Only a few authors have explored using dynamic programming (DP) for scheduling multiple crops. Bright (1986) used constrained differential DP for multi-crop scheduling instead of standard SDP since (at that time) the computational demand of standard SDP was too great. However, some significant simplifications (compared with standard SDP) were required to meet the conditions of optimality for constrained differential DP. Sunantara and Ramirez (1997) used a different method – decomposing multi-crop scheduling into a series of individual single-crop problems. The difficulty with their approach is optimality cannot be guaranteed when there are both system capacity and seasonal water use constraints (Chapter 4).

9.2.2 Proposed SDP algorithm

Since Bright's comment in the early 1980's about computational limitations, computer capacity has increased by several orders of magnitude. This massive increase in computational capacity allowed Córdova and Bras's standard SDP single-crop irrigation scheduling algorithms to be extended to multiple crops (where crop is synonymous with Paddock), thereby allowing some limitations of existing multi-crop SDP irrigation schedulers to be overcome. Computational limitations are still a major problem with this new algorithm. Running the algorithm for only three crops takes hours to days, while four crops would take days to months. Deep soils are more computationally demanding than shallow soils. A comparison between the assumptions made for the proposed SDP irrigation scheduler and previous DP schedulers is given in Table 9-1. A summary of the scheduler is given in Equation 9-1, with further details given in the electronic copy of the source code appended.

Backwards Recursive Equation (after Bellman, 1957):

FOR $t=N_d$ TO $t=0$:

$$B_{t,s}^* = \begin{cases} \text{Max} (R_{N_d,s,I_d}) & t = N_d \\ \text{Max} (R_{t,s,I_d} + B_{t+1,s'}^*) & t \leq N_d - 1 \end{cases} \quad [\text{a}]$$

Cost function:

$$R_{t,s,I_d} = \sum_{p=1}^{N_p} \left(\frac{y_p}{N_d} k_{y_t} E[k_s] \right)_p + c_2 N_{I_d} \quad [\text{b}]$$

and

$$E[k_s] = \frac{\theta_{pv} \cdot \theta_{mid}}{e_p} \quad (\text{where } \theta_{pv} \text{ is at the end of day } t) \quad [\text{c}]$$

Where:

$B_{t,s}^*$ = Optimal expected net return between day t and the end of the irrigation season, given state variables s at the start of day t

R_{t,s,I_d} = Expected net return for day t , given state variables s (at the start of the day), and irrigation decision I_d

t = Day of the irrigation season

s = State variables, which include:

- a) Soil moisture [discretisation interval] for each paddock (Section 7.5.4)
- b) Remaining [seasonal] water (v)

s' = State variables at the start of day $t+1$ given I_d

- a) Soil moisture from Equation 7-9

- b) Remaining water at t (v_t) = v_{t+1} – water use given I_d

I_d = An irrigation decision about which paddocks to irrigate

I_d^* = Optimal irrigation decision, given s and t

N_p = Number of paddocks

N_d = Number of days in the irrigation season

c_2 = Cost of irrigation event (\$/ha)

N_{I_d} = Number of irrigation events given I_d

And for each paddock (p) and day (t):

$$y_p = (\text{Potential yield value from } t = 1 \text{ to } N_d) / \sum_{t=1}^{N_d} k_{y_t} (\$/\text{ha})$$

k_{y_t} = Yield reduction factor (unitless) as a function of t

$E[k_s]$ = Expected value of the water stress reduction coefficient (unitless)

θ_{pv} = Soil moisture probability vector (Section 7.5.2)

θ_{mid} = Soil moisture state discretisation vector (mid-point) (Equation 7-11)

Equation 9-1: Summary of the Backwards Recursive Equation and cost function for the proposed multi-crop SDP irrigation scheduler.

Table 9-1: Comparison between existing DP irrigation schedulers and the proposed SDP irrigation scheduler.

Assumptions	Irrigation scheduling method			
	(Córdova and Bras 1979)	(Bright 1986) Single-crop SDP	(Bright 1986) Differential DP	Proposed multi-crop SDP
No. of crops or paddocks	Single		Multi	
Precipitation	Compound Poisson ¹		No rain	Compound Poisson ¹
Infiltration	Uniform and equal to storm depth (no run-off)			
Irrigation application efficiency	Function of (moisture deficit/ application depth) ²	Function of the moisture deficit to application depth ratio and the spatial application uniformity		Function of (moisture deficit/ application depth) ³
Irrigation depth	Variable			Fixed
Soil moisture storage	Single ‘bucket’ soil layer (Section 2.3). Spatially uniform soil properties and soil moisture.			
Drainage	Linear ⁴ for $\theta > \text{TAW}$. Linear (but reduced rate) for $\text{RAW} < \theta < \text{TAW}$. None for $\theta < \text{RAW}$.	Instantaneous drainage for moisture about field capacity. No drainage below field capacity.		
Reference ET	Deterministic. Mean reference ET as a function of the time of year.			
Potential ET	Single-crop coefficient (proportional to reference ET)			
Water stress coefficient k_c as a function of soil moisture	Piecewise linear, as per Allen et al. (1998).	Non-linear (Bright 1986, Figure 3-9), after Heiler (1981).		Piecewise linear, as per Allen et al. (1998).
Daily event order	(1) Irrigation (2) Drainage & ET (3) Rain	(1) Irrigation & drainage (2) ET (3) Rain & drainage	(1) Irrigation & drainage (2) ET	(1) Irrigation & drainage (2) ET (3) Rain & drainage
Irrigation event cost	Fixed and variable component		Variable component	Fixed component
Yield return	Linear function of actual/potential ET, as per Doorenbos et al. (1979).			
System capacity	Unconstrained		Constrained	
Seasonal water use	Constrained		Unconstrained	Constrained
(1) Stochastic. Poisson storm arrival, Gamma depth distribution (Section 3.5.4). (2) Uniform application depth assumed (3) Bright’s application uniformity efficiency model was allowed for, but was not used in the case study comparisons. (4) Rate proportional to soil moisture (θ). TAW = Total [plant] Available Water. RAW = [Plant] Readily Available Water.				

In contrast to the SA scheduler in Chapter 6, the proposed SDP scheduler optimises individual decisions rather than general management strategies. As a consequence, spatial allocation of water for the SDP scheduler is at a Paddock rather than a Block scale (see Table 2-2 for definitions of Paddock and Block).

While the proposed SDP multi-crop algorithm is based on Córdova and Bras's algorithm, some of the details of the two algorithms are not identical. In particular, irrigation depths were fixed, in line with the restriction of the thesis scope given in Section 2.2.3, and because allowing for variable irrigation depths causes some complications when system capacity is constrained and when scheduling is between more than one crop. Also, drainage of soil moisture above field capacity is instantaneous (in contrast to Córdova and Bras), which is in line with most simple water balance models. However, it is possible to extend the algorithm to include non-instantaneous drainage above soil moisture by using the same Probability Transition Matrices as Córdova and Bras.

One particular limitation of the proposed SDP algorithm is that there is no value given to yield outside of the irrigation season. Of the two crops (pasture and wheat) used in the case studies later in this chapter, this issue affects only pasture; for the case studies that involve wheat, the irrigation season extends from wheat planting to harvest. A solution to this limitation would be to give a dollar value to the soil moisture state at the end of the irrigation season. However, this was not implemented.

9.2.3 Computational times and case study selection

In Equation 9-1, the high number of state variable permutations can result in unmanageable processing time. Therefore, an estimate of computational times (Equation 9-2) was required prior to running the scheduler, to identify case studies that were computationally infeasible. This equation is simplified from the coded version by excluding minor terms, and is therefore valid only for computational times greater than one hour. The full version of this equation may be found in the electronic source code appended.

$$T \sim \left(\sum_{i=1}^{N_T} \left(\prod_{j=1}^{N_P} m_{(i,j)} \right)^2 \right) \left(\sum_{i=1}^{N_I} \text{Bn} \left(\begin{matrix} N_P \\ i \end{matrix} \right) + 1 \right) (c_1 + 1) N_P N_T c_2$$

Where:

T = Computational time (hr)

N_T = Number of Time Aggregation Periods (TAP) per season

N_P = Number of paddocks

N_I = Number of irrigators

m = Number of state discretisation intervals for TAP i and paddock j

$\text{Bn} \left(\begin{matrix} n \\ r \end{matrix} \right)$ = Binomial coefficient nCr (n choose r)

c_1 = Maximum number of irrigation events for the farm, over the entire season

c_2 = 2×10^{-11} (hrs) (P4, 2.8 GHz, 1 GB RAM)

Equation 9-2: Computational time for the proposed multi-crop SDP irrigation scheduler (valid for computational times >1 hour).

$$\left(\prod_{j=1}^{N_P} m_{(i,j)} \right)^2$$

is the most important term in Equation 9-2. This term increases exponentially with an increasing number of paddocks, and becomes excessively large for more than three paddocks, or for large values of m . This holds true even if an improved implementation increased efficiency by one to two orders of magnitude. Memory usage for this scheduler was in the order of 10 MB. Therefore, computational feasibility is limited by computational times rather than memory restrictions. The main factors that affect $m_{(i,j)}$ are:

- Mean reference ET as a function of the time of year;
- ET crop coefficient as a function of time since planting – dictated by crop type and planting date;
- Timing and duration of the irrigation season;
- Rooting depth as a function of time since planting – dictated by crop type and planting date;
- Soil water holding capacity.

In general (for Christchurch climate), for single-crop scheduling, all soil depths and crop types typical of the Canterbury Plains region result in low computational times. For two-crop (or paddock) scheduling, the algorithm is only computationally feasible for shallow to moderately deep soils. For three-paddock scheduling, the only crop that can be reasonably modelled is pasture, with a highly restrictive irrigation season (November to February) and with very shallow soils. Scheduling for more than three paddocks is computationally infeasible. Table 9-2 provides estimated computational times and how these are affected by the crop type, the duration of the irrigation season, and the maximum TAW (where maximum TAW is the Total Available Water at maximum rooting depth). All studies assume climate parameters derived from Christchurch airport data (as described in Chapter 4), one irrigator, and a TAP of four weeks (when all paddocks are pasture) or two weeks (when one or more

of the paddocks is wheat). The irrigation season for wheat (29 September to 15 February) begins two days before planting and ends at harvest. The four highlighted scenarios (1, 2, 4 and 8) have been used as case studies later in this chapter.

Table 9-2: Estimated calculation times for the proposed SDP scheduler illustrating the effect of the number of paddocks, the crop type, TAW and the irrigation season on the computational demand.

ID	No. of Paddocks	Crop types	Max. TAW ⁽¹⁾ (mm)	Irrigation season	c_1 (Equation 9-2)	Comp. time ⁽²⁾ (hr)
1	1	Pasture	[80]	15 Sep-29 Mar	25	0.001
2	1	Wheat	[120]	29 Sep-15 Feb	25	0.002
3	2	Pasture ×2	[80, 80]	15 Sep-29 Mar	75	2.1
4	2	Pasture & wheat	[80, 120]	29 Sep-15 Feb	75	1.9
5	3	Pasture ×3	[80,80,80]	15 Sep-29 Mar	75	5,200
6	3	Pasture ×3	[80,80,80]	01 Nov-20 Feb	75	340
7	3	Pasture ×3	[50,50,50]	01 Nov-20 Feb	75	22
8	3	Pasture ×3	[40,50,60]	01 Nov-20 Feb	75	21
9	4	Pasture ×4	[40,40,40,40]	01 Nov-20 Feb	75	1,500
(1) Total Available Water at maximum rooting depth						
(2) Computation time (P4, 2.8 GHz, 1 GB RAM)						

Scenario ID's 1 and 2 in Table 9-2 show that computational times are very low for single-crop scheduling. Scenarios 1, 3 and 5 highlight the exponentially increasing computational time when scheduling between an increasing number of paddocks. For these scenarios, each additional paddock increases the computational time by a factor of about 2,000. Scenarios 5, 6, and 7 show how decreasing either the duration of the irrigation season or TAW decreases computational time. Scenario 9 shows how even for very shallow soils and a very restricted irrigation season, computational times become infeasible for four or more crops.

9.3 Case Study Parameters

Two crops were modelled – pasture and wheat. Pasture was modelled with a constant soil moisture storage (rooting depth) and an ET crop coefficient of 1.0. Wheat was chosen as a representative cereal. Wheat parameters, as a function of the time of year, are presented in Figure 9-1. Wheat parameters were obtained from Table 12 of Allen et al. (1998), and modified according to typical summer planting and harvesting dates for Canterbury.

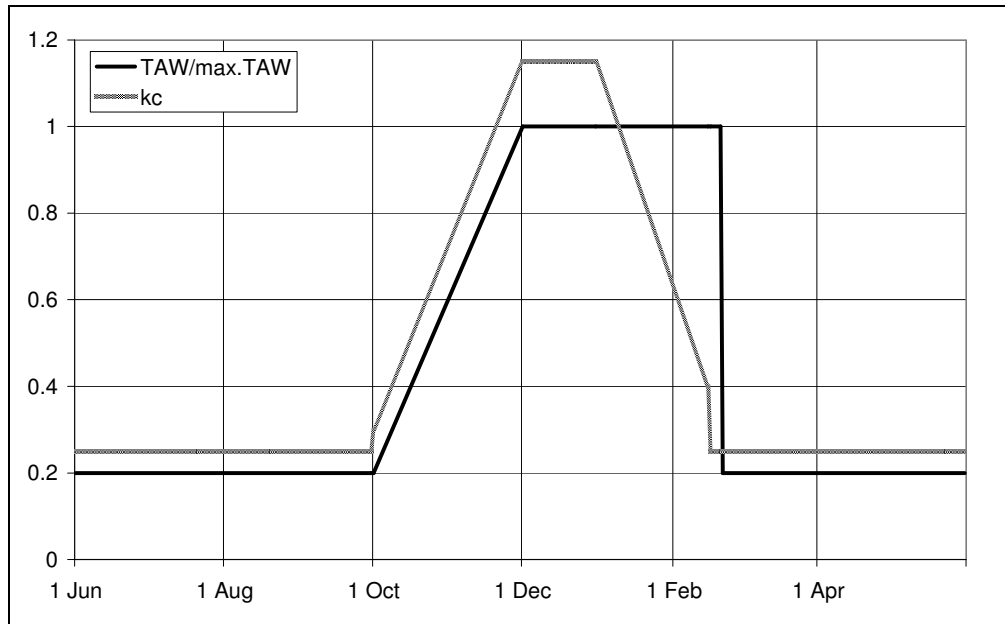


Figure 9-1: Crop coefficient (k_c) and soil moisture storage (TAW/max.TAW) series for wheat.

Climate parameters were derived from historic Christchurch airport data (Table 3-1). Rainfall time-series were synthetically generated using the compound-Poisson model (Section 3.5.4); while the annual reference ET time-series was assumed to be the mean of the historic values, as a function of the time of year. Synthetic, rather than historic, climate data was used in both the case study and objective function simulations so that the climate assumptions embedded within the SDP formulation would be exactly true. Ten case study seasons were simulated.

A single irrigator was used for all case studies. Irrigator application depth and seasonal water use limits were chosen so that seasonal limits were always constraining (i.e. required deficit irrigation), and so that system capacity was constraining for two or more paddocks. The low daily application depths could be typical of a centre-pivot system. A paddock area of 10 ha was assumed; therefore 1 mm of applied water per paddock corresponds to 100 m³.

The irrigation season for wheat begins two days before planting and ends at harvest. Two different irrigation seasons were used for pasture. The first, 15 September to 29 March, would be a typical season given moderate seasonal water use limits (where water is unrestricted, irrigation often continues throughout April). The second, 1 November to 20 February, was used for the three-paddock case study in order to make the SDP algorithm computationally feasible. Such a season could be encountered in practice if seasonal water limits were highly constraining. Particular start and end dates of the seasons were chosen so that the length of the season is exactly divisible by the TAP. The TAP was four weeks (when all paddocks were pasture) or two weeks (when one or more of the paddocks was wheat). Potential ET (k_c , et_o) and soil moisture storage (TAW) are functions of time. Within both the Proposed Heuristic and SDP schedulers, these two functions are approximated step-

wise, with the width of a step equal to the TAP. Therefore, a shorter TAP was used when wheat was grown in one of the paddocks, due to the greater time variability of $k_c \times et_o$, and TAW for wheat.

The cost of an irrigation event (c_2 , Equation 9-1) was set to zero and the seasonal water limit sufficiently constraining, such that it was always economic to apply all available seasonal water. Potential yield values (y_p , Equation 9-1) are given in Table 9-3. The yield reduction factor (k_y , Equation 9-1) for both pasture and wheat was assumed to have a constant value of 1.0 throughout the irrigation season. The cost function for the case study and the SA scheduler objective function simulations is given in Equation 9-3.

Table 9-3: Potential annual pasture and wheat economic yields

Crop	Potential yield value (\$/ha)	Source
Pasture – medium production feed supply	12,000 kg DM @ 85% utilisation & \$0.15/kg = \$1,530/ha	Canterbury Agriculture Ltd. (2005)
Premium bread Wheat	6,800 kg @ \$0.306/kg = \$2,080.80/ha	Canterbury Agriculture Ltd. (2005).

$$Y = \sum_{p=1}^{N_p} \left(y_p \sum_{t=1}^{N_d} \left(k_y \frac{e_a}{e_p} \right)_t \right)_p$$

Where:

- Y = Yield value during the irrigation season (\$/ha)
- y_p = Potential yield value (\$/ha) (Equation 9-1)
- N_p = Number of Paddocks (p)
- N_d = Number of days in the irrigation season
- t = Day of the irrigation season
- k_y = Yield reduction factor as a function of t (unitless)
- e_a = Actual evapotranspiration as a function of t (mm)
- e_p = Potential evapotranspiration as a function of t (mm)

Equation 9-3: Cost function for the case study and the SA scheduler objective function simulations

Table 9-4: Case study parameters for the comparison between the SA scheduler and SDP irrigation scheduling

Parameter	Case study 1	Case study 2	Case study 3	Case study 4
General parameters				
Paddock description and Max. TAW (in mm)	1×Pasture (80)	1×Wheat (120)	1×Pasture (80) 1×Wheat (120)	1×Pasture (40) 1×Pasture (50) 1×Pasture (60)
Irrigation depth (mm)	15		8	12
Seasonal water limit (100 m ³)	300	180	480	900
Irrig. season	15 Sep-29 Mar		29 Sep-15 Feb	1 Nov-20 Feb
Irrig. season duration	196 days	140 days		112 days
Time Aggregation Period	28 days	14 days		28 days
SA scheduler parameters				
Max. no. of effective decision variables	6	9	19	11
No. of sample seasons ¹	10			
Initial temp. (% of melting temp.)	25%			
No. of temp. steps	6			
No. of simplex steps per temp. step	50 x (No. effective decision variables)			
Population Analysis used?	Yes		No	
No. of SA runs each optimisation ²	5		3	
(1) Number of sample climate seasons simulated for a single evaluation of the objective function. (2) When Population Analysis was used to isolate and search a macro-depression (Section 8.8), the population size was four and SA was run one additional time to search within the isolated macro-depression.				

Computation times for the SA scheduler are governed by the number of objective function evaluations. For the case studies in Table 9-4 the computation times ranged from 2-8 hours²² (on a P4 computer) per case study season.

9.4 Results

Figure 9-2 to Figure 9-5 present the results from the four case studies. For each of the case studies, 10 different seasons of synthetically-generated climate data were simulated (these 10 seasons are different from the 10 sample seasons used in the objective function of the SA scheduler). Table 9-5 collates and summaries the results from these four figures. Three different scheduling methods are compared: the SA scheduler from Chapter 6, the proposed SDP scheduler, and a method termed Constant TSML (trigger soil moisture levels). Constant TSML is a method of scheduling that makes use of the Markov Chain Water Use Equation from Chapter 7, but does not use any optimisation procedure. Farm

²² Case study 1: 2.1 hr; Case study 2: 1.3 hr; Case study 3: 6.8hr; Case study 4: 8.2 hr

Irrigation Strategies are calculated so that expected TSMLs are constant for each Paddock and TAP. This method of scheduling is expected to be comparable, or better than, current best management practice. It provides another benchmark from which to compare the two optimisation methods.

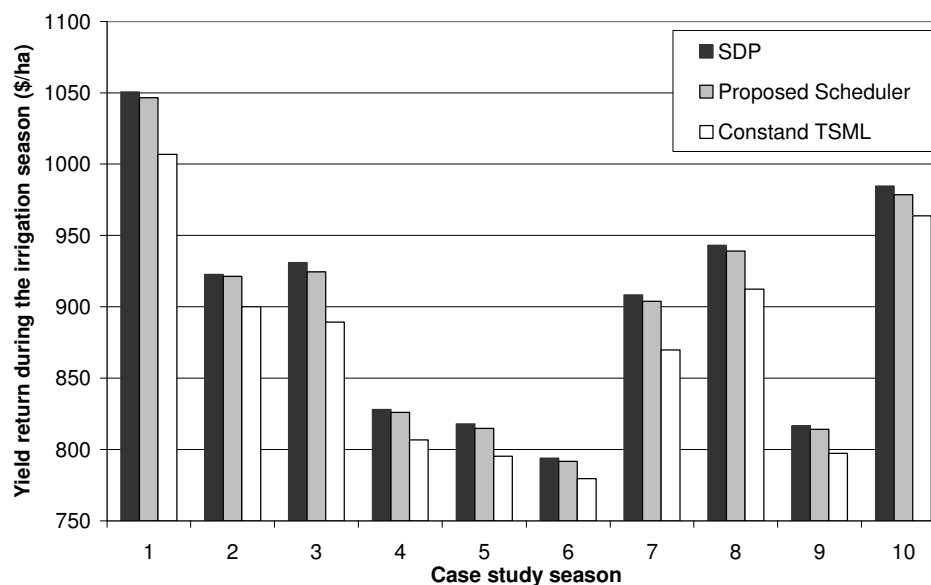


Figure 9-2: Scheduling irrigation using SDP, the SA scheduler, and the Constant TSML method. Case study 1 (1×Pasture, TAW=80 mm) – Seasonal water use limit = 30,000 m³ over 10 ha

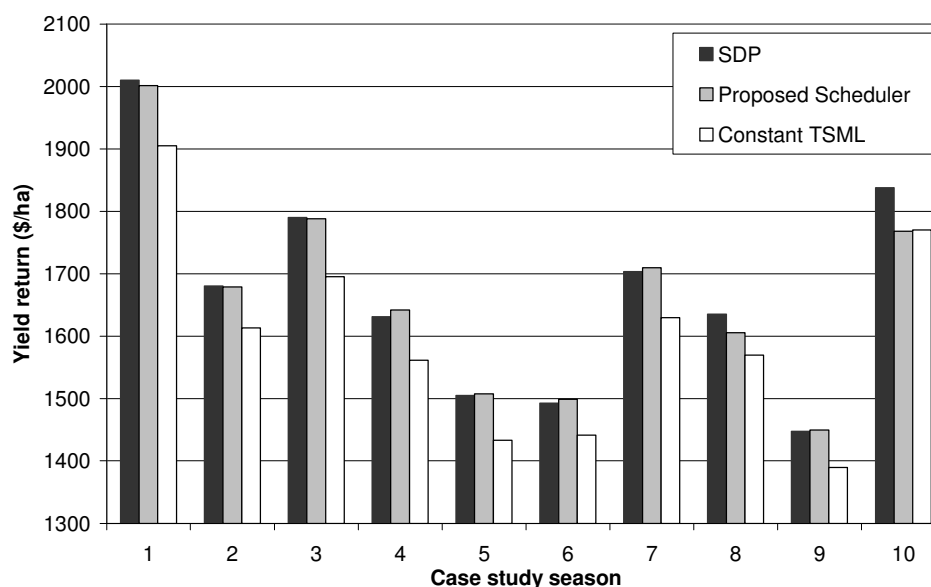


Figure 9-3: Scheduling irrigation using SDP, the SA scheduler, and the Constant TSML method. Case study 2 (1×Wheat, TAW max.=120 mm) – Seasonal water use limit = 18,000 m³ over 10 ha (for comparison, the potential yield from full irrigation was \$2,080)

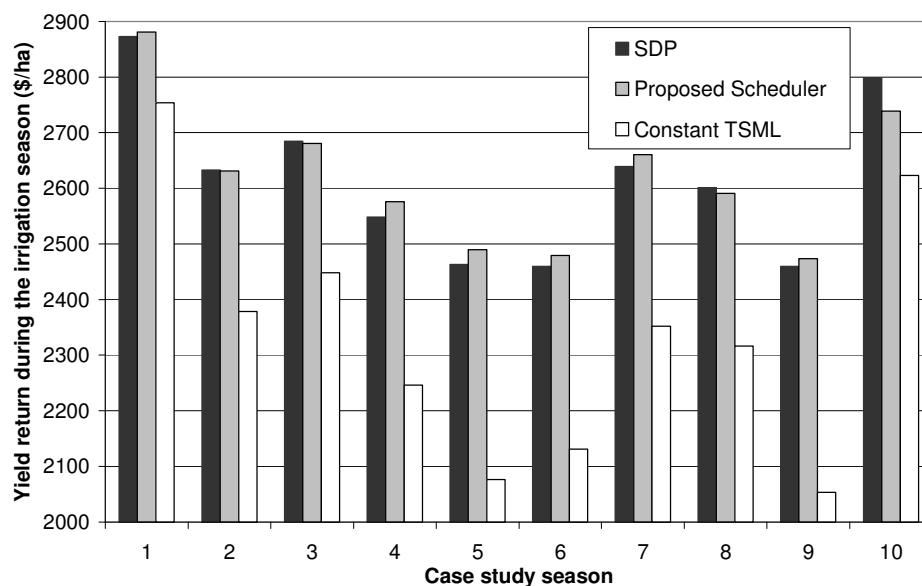


Figure 9-4: Scheduling irrigation using SDP, the SA scheduler, and the Constant TSML method. Case study 3 (1×Pasture, TAW=80 mm; 1×Wheat, TAW max.=120 mm) – Seasonal water use limit = 48,000 m³ over 20 ha

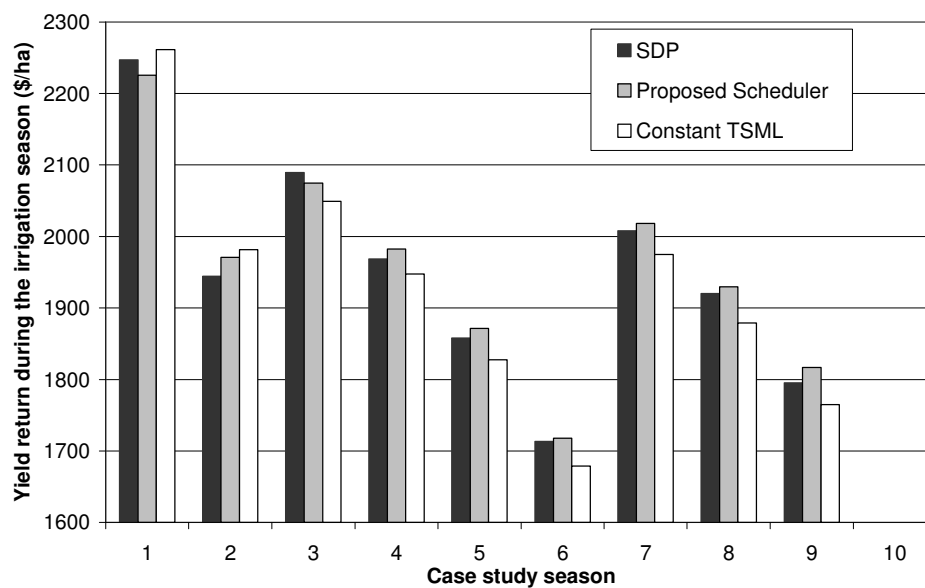


Figure 9-5: Scheduling irrigation using SDP, the SA scheduler, and the Constant TSML method. Case study 4 (3×Pasture, TAW=[40,50,60] mm) – Seasonal water use limit = 90,000 m³ over 30 ha

Table 9-5: Average net return (as a % of the average net return from scheduling using SDP), demonstrating the ability of the SA scheduler to closely match the performance of SDP.

Scheduling method	Case study				Averaged over all case studies
	1	2	3	4	
SDP	100.0%	100.0%	100.0%	100.0%	100.0%
SA scheduler	99.6%	99.5%	100.2%	100.3%	99.9%
Constant TSML	96.9%	95.7%	89.4%	99.0%	95.3%

Figure 9-2 to Figure 9-5 show that, for any given case study season, the particular scheduling method that results in the greatest yield value is probabilistic. Generally, the SA scheduler and the SDP resulted in comparable performance. However, occasionally, the Constant TSML method would result in the greatest yield value. This observation can be explained, since both the SDP and the SA scheduler seek to optimise the *expected* future yield return. However actual performance for a given season is probabilistic, due to the unknown [stochastic] future climate.

A paired *t*-test was used to test the null hypothesis the mean performance of the three methods were identical. The percent differences (compared with SDP scheduling) for the SA scheduler and Constant TSML had a mean and standard deviation of -0.08% and 0.96%, and -4.90% and 4.24% respectively. All data for the four case studies were aggregated, resulting in a sample size of 39²³. The resulting *t* statistic was 0.53 and 7.2 for the SA scheduler and the Constant TSML, respectively²⁴. The critical value of *t* was 1.3 and 3.6 for a 10% and 0.05% (respectively) probability of rejecting the null hypothesis. Therefore, it may be concluded with some confidence that the SA scheduler and the SDP scheduling methods have similar mean performance, while scheduling using Constant TSML results in inferior mean performance.

²³ That is 39 case study seasons

²⁴ $0.0008/(0.0096 \sqrt{39})=0.53$; $0.049/(0.0424 \sqrt{39})=7.2$

9.5 Conclusion

The SA scheduler was able to closely match the performance of SDP, suggesting the following three assumptions listed earlier in this chapter are reasonable:

- 1) That the method of automatic adjustment (within objective function simulations) of irrigation strategies, in response to deviations between expected and simulated water use (described in Section 6.5), is a reasonable approximation of the optimal adjustment strategy.
- 2) That the restricted range of irrigation schedules (as a consequence of decision variables describing general management decisions) still allows for close to optimal scheduling (Section 6.4).
- 3) That the custom heuristic method (Chapter 8) can produce near optimum solutions, given sufficient calculation time.

The hypothesis is made that the above assumptions will also be reasonable when complex farm models are incorporated into the SA scheduler. This hypothesis is partially tested in Chapter 10.

10 CASE STUDIES OF CANTERBURY PASTORAL FARMING

10.1 Context and Overview

This chapter demonstrates the ability of the SA scheduler to incorporate complex farm system models. In Chapter 4, over-simplistic crop models were identified as the main limitation of existing optimal irrigation scheduling methods. Chapter 6 proposed a heuristic method for scheduling that could incorporate complex farm simulation models within the objective function. In Chapter 2, FarmWi\$e (by CSIRO Plant Industry) was identified as a complex model, well suited for modelling many NZ agricultural systems; this model is also suitable for use within the SA scheduler. This chapter presents a case study comparing the simulated yield return when scheduling irrigation using: (a) the SA scheduler; (b) a method termed Constant TSML; and (c) current best management practice methodology. The FarmWi\$e model is used for the case study simulation, and is also embedded within the SA scheduler's objective function. The case study farm operation focused on growing and cutting grass (e.g. for dairy support). Both the quantity and quality of grass was considered in the cost function.

This chapter reiterates the advantages of using FarmWi\$e in the SA scheduler (Section 10.2), describes the case study (Section 10.3), and how FarmWi\$e was embedded in the SA scheduler (Section 10.4). Preliminary comparisons showed good potential for FarmWi\$e to model well the quantity and quality of pasture production on the Canterbury Plains (Section 10.5). The custom heuristic method demonstrated convergence to some localised minima that were significantly better than solutions found by random sampling, even with rapid SA cooling (Section 10.6). Case study results in Section 10.7 found that the SA scheduler out-performed other scheduling methods, as hypothesised in Chapter 6. For this particular case study, the SA scheduler was able to increase the mean yield return compared with current irrigation best management practice, by 10% (while using the same amount of seasonal water); or alternatively expressed, decrease seasonal water use by 20-25% (while still producing the same mean yield return).

10.2 FarmWi\$e

As part of its GrazPlan programme, CSIRO Plant Industry has produced a series of analysis and decision support tools for temperate Australia pastoral farming systems. FarmWi\$e is the most recent and generalised tool within the GrazPlan programme, and provides a generic modelling environment for simulating many different farming operations. CSIRO Plant Industry's Common Modelling Protocol (CMP) framework allows for the addition and/or interchanging of third party model components. In particular, a partnership with the (Australian) Agricultural Production Systems Research Unit has made available a large number of cropping models. Therefore FarmWi\$e has the

capacity to model a wide range of NZ farming operations including dairying, sheep and beef, and mixed cropping, the primary users of irrigation water in NZ. Advantages of using FarmWi\$e for modelling farm profit (particularly pastoral systems) within the SA scheduler includes the following:

- The model is applicable under both dryland and irrigated conditions.
- The model has been field-verified for farms that are similar to typical NZ pastoral systems.
- There are peer-reviewed publications of model components.
- There is a track history of model components and related decision support tools having widespread acceptance and use by the farming industry (in Australia).
- The model has modern computer engineering architecture, allowing efficient implementation of a custom irrigation control component. In particular, the model has built-in functionality for copying the simulation state while the simulation is running.

Further details of FarmWi\$e, together with alternative models considered, are presented in Section 2.4.

10.3 Case Study Description

The irrigation scheduling case study farm operation focused on growing and cutting grass (e.g. for dairy support) for a single Paddock.

10.3.1 Farm system components

The FarmWi\$e model has been built as a collection of inter-changeable components which communicate with one another via a set of defined messages. This allows any component which conforms to CSIRO's CMP to be used in FarmWi\$e. This protocol is computer language independent, allowing components written in various different languages to be used within the same model. The graphical user interface used for constructing and parametising the case study simulation is illustrated in Figure 10-1. This interface allows components from a palette to be dragged and dropped into a tree representation of the farm setup, allowing rapid model construction. For the FarmWi\$e model embedded within the SA scheduler, the construction and running of simulations was via direct Protocol communication.

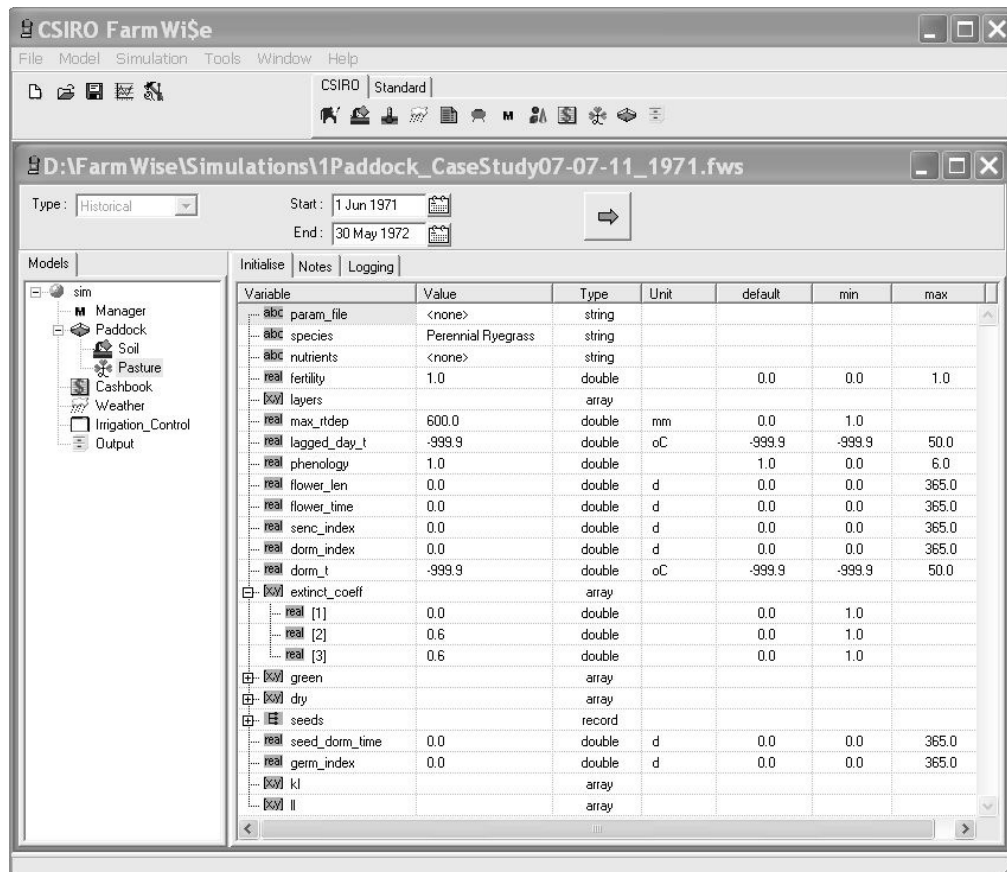


Figure 10-1: FarmWi\$e model structure and parameterisation input console for the case study farm

The irrigation scheduling case study farm used in this chapter was comprised of the follow components: (a) a paddock; (b) a soil (a sub-component of the paddock); (c) pasture (a sub-component of the paddock); (d) climate timeseries; (e) a manager for controlling the timing of pasture cuts and scheduling irrigation (when using the best management practice method); (f) a custom irrigation controller for scheduling irrigation (when using the SA scheduler or Constant TSML methods); (g) a financial calculator; and (h) a component for handling the recording of outputs. These components are described in more detail below. The financial calculator is described in Section 10.3.3 and the custom irrigation scheduling component in Section 10.4. The recording of outputs required no parameter setting and is not discussed. Electronic copies of model input files (in XML format) are appended.

The main inputs for the paddock component are the land surface slope (set to zero) and the area (set to 60 ha).

The soil component principally models the dynamics of water movement. The model allows for multiple soil layers, non-instantaneous downward percolation, and a vertical non-homogeneous rate of moisture removal by evaporation and transpiration. Parameters used in the case study are presented in Table 10-1. Further details of this component is given by Moore et al. (1997). While this particular

component did not model nutrient dynamics, a soil component does exist for FarmWi\$e, which models the transport and transformation of nitrogen, phosphorus and sulphur.

Table 10-1: Soil component parameters

Parameter	Value	Comment
Layer thickness	600 mm	Single-soil layer. Thickness equal to rooting depth.
Water content at saturation	0.47 mm/mm	
Water content at field capacity	0.25 mm/mm	
Water content at wilting point	0.10 mm/mm	
Total (plant) Available Water	90 mm	Given a rooting depth of 600 mm
Bulk density	1.3 Mg.m ⁻³	
Saturated hydraulic conductivity	500 mm/d	
Clay content	20%	
Sand content	70%	
Soil albedo	0.17	
Evaporation rate parameter α	3.5 mm.d ^{-0.5}	See Moore et al. (1997), Table 10.

While the soil component allows for multiple soil layers, a single layer was used for the irrigation scheduling case study. The reason for this was to increase the agreement between the current version of the Markov Chain Water Use Equation in Chapter 7 (which assumes a single-soil layer) and the FarmWi\$e irrigation water use model. Future work is still required to extend the Water Use Equation by allowing for dual soil layers (Section 7.9).

The pasture component can model a wide range of grass and forbe phenological characteristics (Moore et al. 1997). Many of the model parameters are pre-defined for a given plant species. For the case study, initial values for state variables were obtained by running the model for 12 months, and allowing state variables to converge to values appropriate for the soil and climate conditions. Consequently, only three parameters required setting: the plant species; the maximum rooting depth (although rooting depth would have converged to an appropriate value for the soil conditions if desired); and a soil fertility scalar parameter. The plant species was perennial ryegrass, selected because it is the predominate species sown in NZ. The maximum rooting depth was set to 600 mm, and the soil fertility scalar parameter was set to 1.00 (corresponding to high fertility). The plant nutrient model required only the setting of the fertility parameter. A more detailed nutrient model was available if used in conjunction with the soil nutrient transport and transformation component.

Daily historical climate timeseries were used in the simulation. The climate station was Christchurch Airport (Table 3-1). One millimetre was subtracted from each wet day to account for canopy interception losses (Section 3.5.3). The standard ET data input for FarmWi\$e is pan evaporation, since this is a common parameter available in Australia. Within FarmWi\$e, generally, reference ET is calculated as 0.8 times the pan evaporation. Therefore, in order to increase agreement with the Water

Use Equation, the pan evaporation timeseries FarmWiSe input was calculated as the reference ET timeseries (from Table 3-1) divided by 0.8. Daily minimum and maximum temperatures, vapour pressure, and wind speed for the Christchurch Airport climate station were sourced from NIWA (2007). The historical timeseries used for the case study simulations were June 1961-May 1962, June 1971-May 1972, June 1981-May 1982, June 1991-May 1992, and June 2001-May 2002. The historical timeseries used by the SA scheduler (for the objective function simulations) were September 1960-May 1961, September 1965-May 1966, September 1970-May 1971, September 1975-May 1976, September 1980-May 1981, September 1995-May 1996, and September 2000-May 2001. Consecutive years of climate data were not used so that long-term climate trends could be accounted for, while simulating the minimum number of seasons.

The manager component controls the timing of pasture cuts, and the scheduling of irrigation when using the best management practice method. Pasture is cut when the total cover exceeds 4,000 kg-DM (dry matter)/ha, down to a height of 1,500 kg-DM/ha. This cutting regime is slightly different to standard practice, which would cut from 2,800 down to 1,000 kg-DM/ha (Bywater 2007). The different cutting regime was used to increase the time when the pasture canopy was fully closed (by increasing the cut height and reducing the number of cuts). The reason for this was to increase the agreement between the current version of the Markov Chain Water Use Equation (which assumes a single ET crop coefficient) and the FarmWiSe irrigation water use model (which models evaporation and transpiration separately). Future work is still required to extend the Markov Chain Water Use Equation to model evaporation and transpiration separately via a dual ET crop coefficient (Section 7.9). Irrigation best management practice applied 15 mm when the soil moisture deficit increased above 30 mm, until all available seasonal water was used. The irrigation season was from 15 September to 4 April.

10.3.2 Seasonal water limit

Optimal irrigation scheduling techniques are likely to be of most benefit in NZ when the water available (to a farm) for the season is insufficient for full production. Therefore for the case study, the seasonal water limit was chosen so that deficit irrigation would be required.

Figure 10-2 shows the results from several simulations (but with different seasonal water limits), using the same farm parameters as described in Section 10.3. Irrigation scheduling uses the Constant TSML method – which uses the Markov Chain Water Use Equation to make forward predictions of water use, and schedules irrigation such that the expected TSML is constant for the remainder of the season. These simulations were used to generate a relationship between the yield return and seasonal water use.

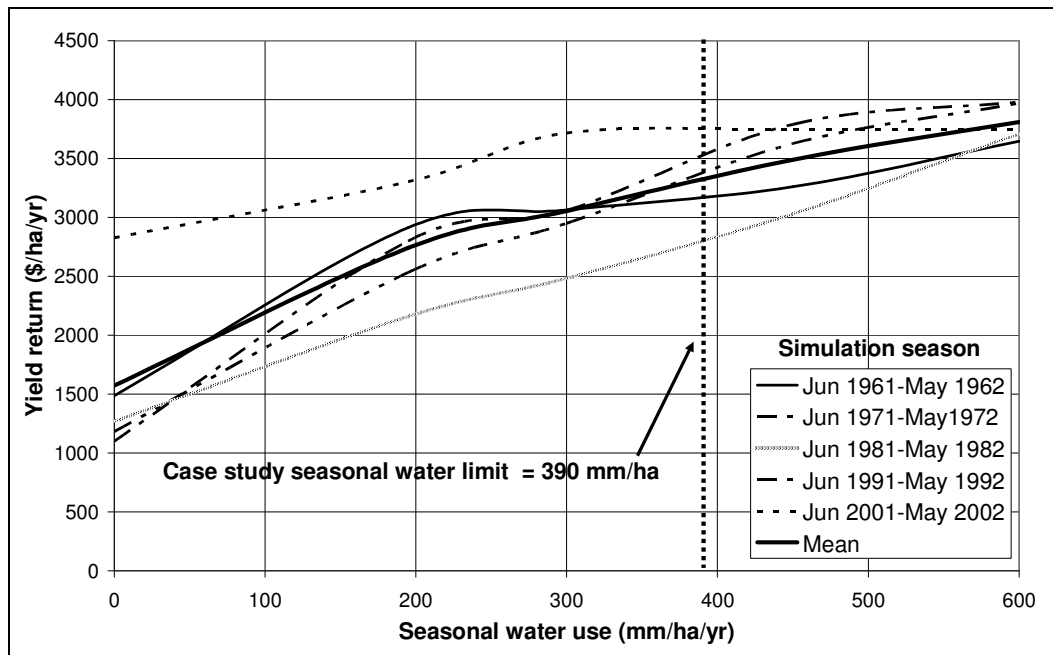


Figure 10-2: Annual yield return as a function of the available seasonal water. Simulation is of cutting grass for dairy support. Irrigation scheduling decisions use the Constant TSML method.

Where Figure 10-2 has a positive gradient, the available seasonal water is insufficient for full production. The steeper the gradient, the greater the expected benefits from using irrigation scheduling optimisation, since the gradient represents the value of an incremental increase in available water. Where this relationship plateaus (e.g. simulation season June 2001-May 2002, for available water greater than 300 mm/ha/yr), there will likely be few benefits from optimal scheduling. For the case study, a seasonal water limit of 390 mm/ha/yr was used, since for most of the case study seasons the available water verses yield return relationship is positive at this point.

10.3.3 Cost function

The FarmWi\$e financial calculator component incorporates the cost function. For the case study, only yield value was considered. The cost of irrigation was assumed negligible so that all seasonally available water was used. The value of the yield is a function both of the quantity and quality of pasture (Equation 10-1). Below a dry matter digestibility (DMD) of 0.5, it was assumed that the pasture had no value but instead would incur a cost associated with harvesting and disposing of the grass. Above this minimum threshold, the value of pasture was assumed to be linearly related to the quality (Bywater 2007).

$$\text{Yield value} = \text{Cut amount (kg-DM/ha)} \times 0.40 (\$/(\text{kg-DM/ha})) \times (\text{DMD}-0.5)/0.5$$

Where: DMD = Average [proportional] dry matter digestibility (0-1.0) (unitless)

Equation 10-1: Yield value (\$/ha) as a function of pasture quantity and quality, for an individual pasture cut.

From Equation 10-1, the value of pasture will generally be in the range of 0.08 - 0.24 \$/kg-DM/ha (for a DMD of 0.6-0.8, respectively). These prices are reasonable for dairy support in Canterbury. The SA scheduler optimises irrigation management strategies such that the average annual yield value (the sum of all pasture cuts) is maximised.

10.4 Custom Irrigation Scheduler Component Implementation

The SA scheduler and Constant TSML scheduling method were implemented within FarmWi\$e by a custom CMP component. Protocol support was principally provided via two modules supplied by CSIRO Plant Industry, Black Mountain Laboratories. One of these modules wrapped the component within a class and managed the .NET/Win32 interface, while the second module provided Protocol functionality (CPI 2007). The custom component communicated with the rest of the simulation via the CMP engine. An electronic copy of source code for the custom component is appended.

Figure 10-3 and Figure 10-4 show how the author's component interacted within the CMP. Component code logic was written in C#, a computer programming language released in 2000 by Microsoft as part of the overall .NET strategy (Schildt 2006). Parameters used in the case study for the custom component are given in Table 10-2.

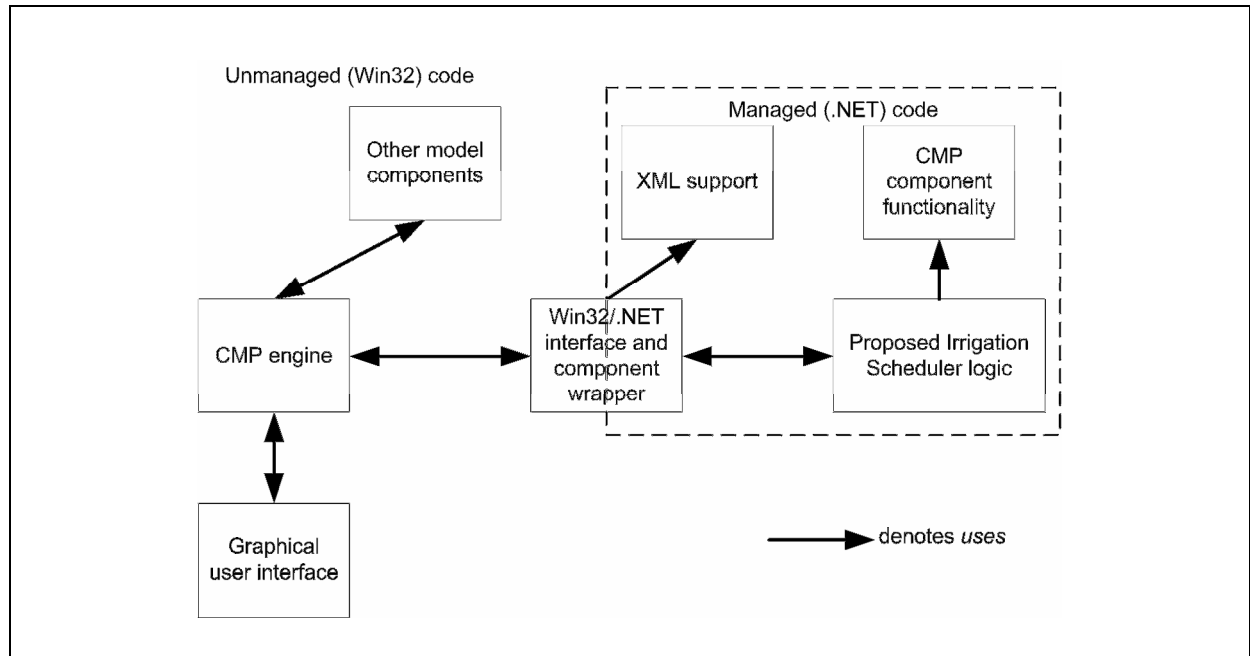


Figure 10-3: Interaction of the custom irrigation scheduling component within the CMP, showing the modules supplied by CSIRO which support the development of custom .NET components.

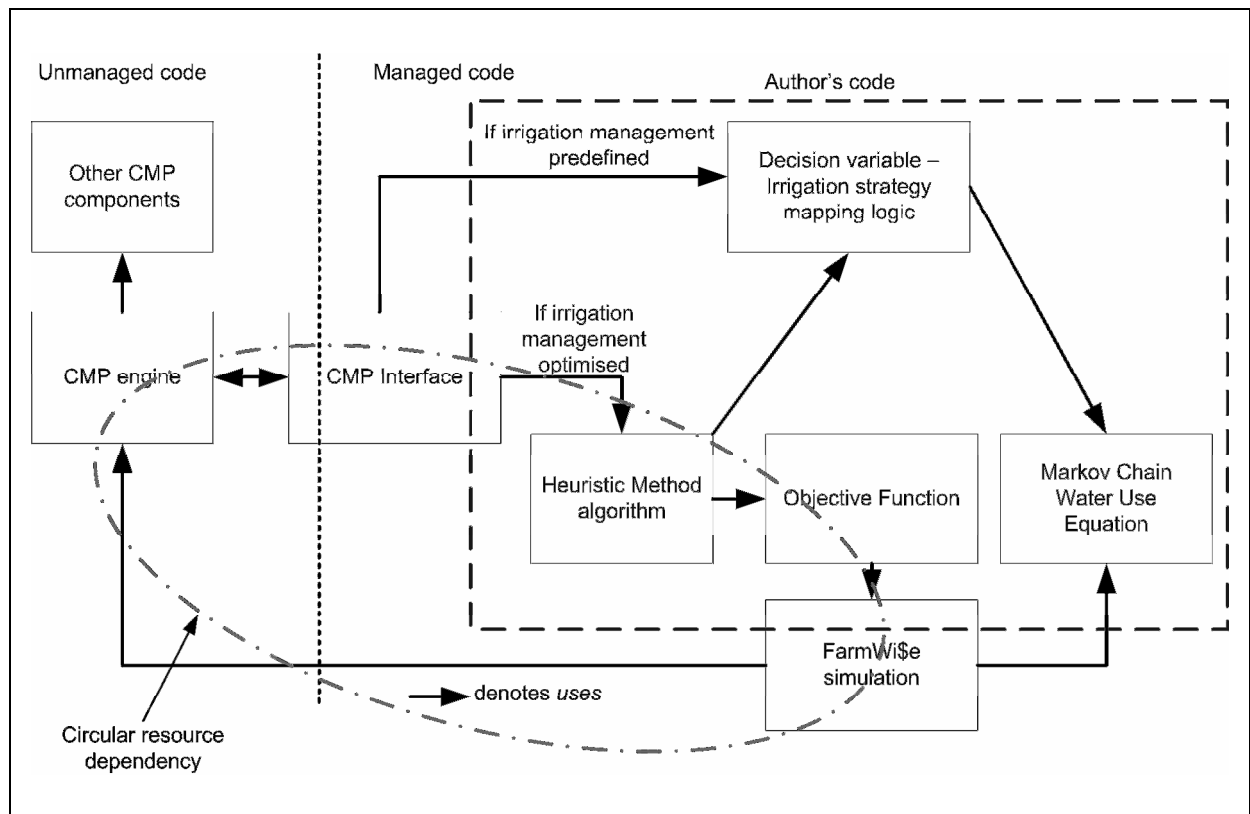


Figure 10-4: Interaction of the custom irrigation scheduling component within the CMP, illustrating circular resource dependency requiring multi-thread processing.

Figure 10-4 shows how circular resource dependency requires that a new processor thread be created each time an objective function simulation is run. One particular implementation difficulty was these processor threads did not always terminate once an objective function simulation was completed. Threads likely fail to terminate due to errors in disposing of one or more objects in the thread. The disposal of a managed (.NET) object when it is unreferenced occurs automatically within managed code. For unmanaged (Win32) code, ensuring objects are appropriately disposed when they are no longer referenced is generally the responsibility of the programmer. Interfacing between managed and unmanaged environments is particularly error-prone, since either environment has a full knowledge or control of the other environment. Figure 10-4 illustrates that unmanaged components use managed components, which in turn use the same unmanaged components multiple times (but each time with a different processor thread). Therefore, it is likely the problem of threads sometimes failing to terminate is associated with the Win32/.NET interface and component wrapper module. Future software version releases from the FarmWi\$e developers are expected to fix this bug.

The problem of processor threads not always terminating, limited the maximum number of objective function simulations for each optimisation run to about 4000. Each evaluation of the objective function simulated nine different seasons; therefore, the maximum number of objective function evaluations per optimisation run was only 440. For comparison for case studies in Chapter 9, each optimisation run used up to 18,000 objective function evaluations. In order to use SA to optimise with

such a small number of objective function evaluations, dimensionality had to be reduced as much as possible by minimising the number of Blocks and TAP. Consequently, scheduling was only for one Block and the season divided into four TAP, resulting in an effective dimensionality of three. For simplicity the Block was assumed to contain only one Paddock (therefore for this case study, Block and Paddock are synonymous). In future work, once the thread termination issue has been resolved, multi-Block scheduling using FarmWiSe will be possible, just as multi-Block scheduling using a FAO 56 crop model was done in Chapter 9.

To reduce the case study computational demand, the custom heuristic method was run once (to obtain a new Optimal Farm Irrigation Strategy) at the start of each TAP. Each other day of the TAP this Irrigation Strategy was adjusted in response to deviations between expected and actual water use, as described in Section 6.5. This is in contrast to the case studies in Chapter 9, where the heuristic method was run each day of the irrigation season. With this change computational times for the SA scheduler were about 8 hours per case study season, on a P4 computer.

Table 10-2: Custom irrigation control component case study parameters

Parameter	Value	Comment
<i>Irrigation System</i>		
Irrigation season	15 Sep – 4 Apr	200 days
Time Aggregation Period (TAP)	50 days	4 TAPs per season
Seasonal water use limit	390 mm/ha	See Section 10.3.2
Irrigation depth	15 mm	
<i>Markov Chain Water Use Equation</i>		
Total available water (TAW)	80 mm	10% less than the TAW within the FarmWiSe model, to counter the differences between the two ET models
RAW/TAW ratio	0.2	20% less than the ratio within FarmWiSe, to counter the differences between the two ET models
Evapotranspiration parameters	As per Chapter 3	
Rainfall parameters	As per Chapter 3	
<i>SA scheduler</i>		
Initial temperature/ melting temperature ratio	0.3	
Number of temperature steps	3	
Steps/temperature step	20 x (No. decision variables-1)	
Objective function historic climate timeseries.	1965, 1970, 1975,...2000 irrigation years	To avoid an unfair advantage to the SA scheduler (compared with other scheduling methods) there is no overlay between the climate timeseries used for the case study, and within the objective function.

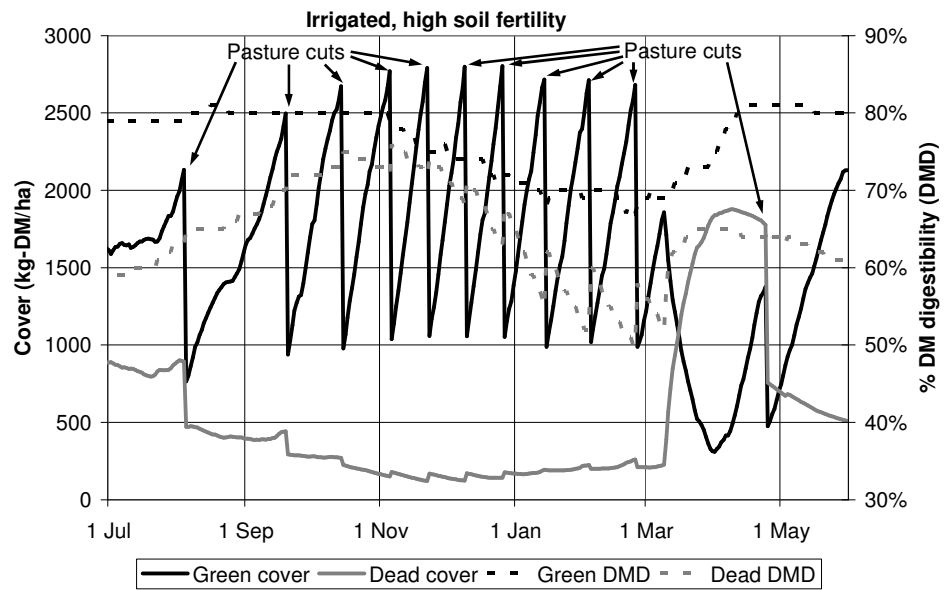
10.5 FarmWi\$e Verification for New Zealand conditions

Preliminary model verification of FarmWi\$e for pastoral farming on the Canterbury Plains was undertaken by a series of simulation run from June 1998 to May 2004. Christchurch historical climate (Table 3-1) was used and pasture parameters were as described in Section 10.3. In addition to the single-layer soil used for the case study, a multi-layer soil was also modelled. Irrigation used the same best management practice as the case study, but without any seasonal water limit or start and end date for the irrigation season. The pasture cutting regime differed from the case study to more closely model standard practice; pasture was cut when the total grass cover was 2,800 kg-DM/ha down to 1,000 kg-DM/ha.

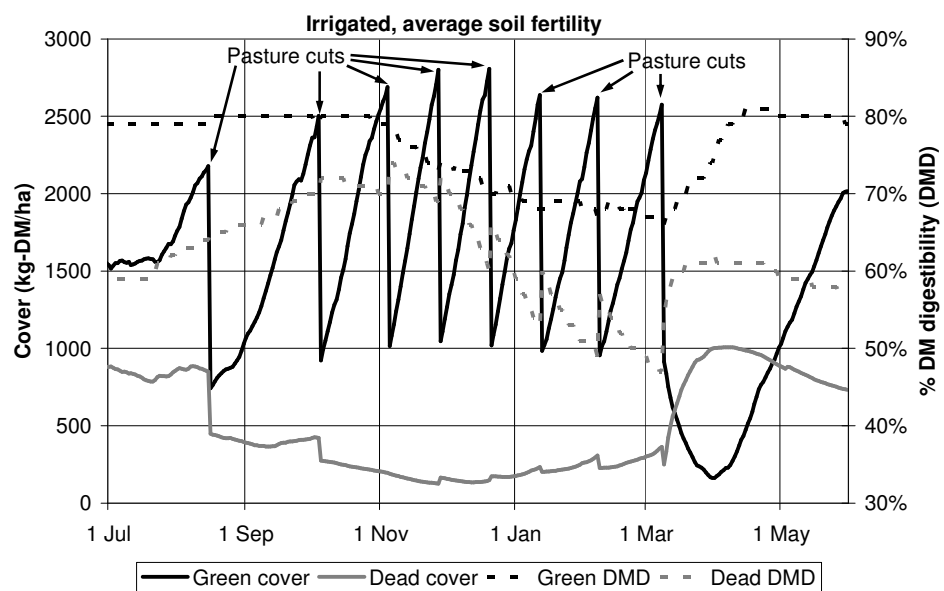
Table 10-3 presents simulated annual production, illustrating how pasture quality and quantity are functions of soil fertility, the number of layers within the soil model, and whether the paddock is irrigated or dryland. Figure 10-5 illustrates the simulated time-varying natural of pasture growth and quality variations for the period from July 1998 to May 1999. Within the model, grass tissue age is grouped into four pools: green established, green senescing, dead standing, and dead litter. For this figure, the established and senescing pool were grouped together, and the dead standing and litter were grouped together.

Table 10-3: Perennial ryegrass quantity and quality (as modelled by FarmWi\$e) given Christchurch climate from June 1998-May 2004, TAW = 90 mm, and pasture cuts when the total cover is 2,800, cut down to 1,000 kg-DM/ha.

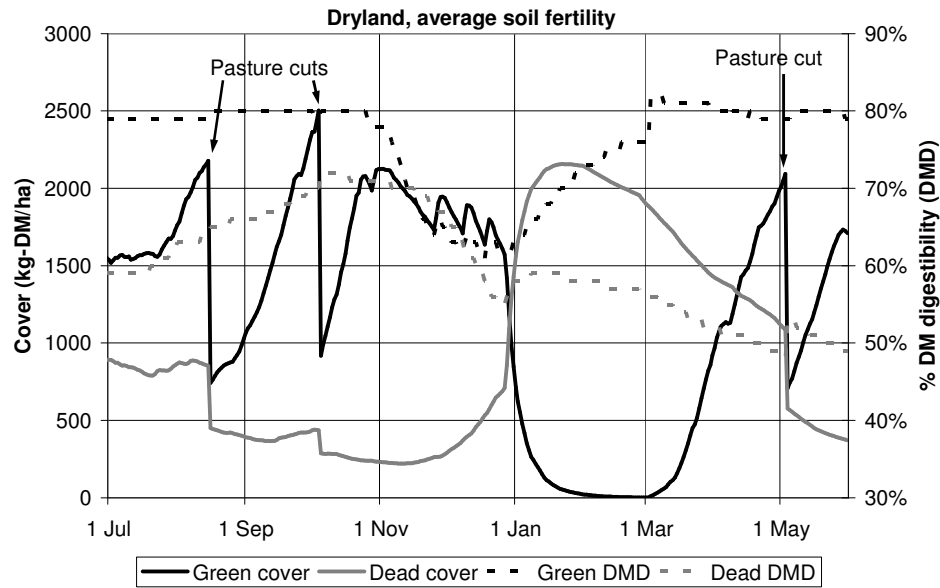
Run ID	Irrigated?	Soil fertility ¹	Annual DM production (kg-DM ³ /ha)		Mean DMD ³
			Range	Mean	
Single-layer soil					
1	No	High	9,000-12,600	10,800	76%
2	No	Average	5,400-9,000	6,840	75%
3	Yes	High	19,800-21,600	20,520	73%
4	Yes	Average	14,400-16,200	15,120	71%
Multi-layer soil					
5	No	High	7,200-10,800	8,640	75%
6	No	Average	5,400-7,200	5,760	74%
7	Yes	High	19,800-21,600	20,520	73%
8	Yes	Average	14,400-16,200	15,120	71%
Notes:					
(1) High and average soil fertility corresponds to a value of 1.00 and 0.75 for the fertility scalar parameter respectively.					
(2) Production values ignore any harvesting losses during cutting and bailing.					
(3) Dry matter digestibility.					



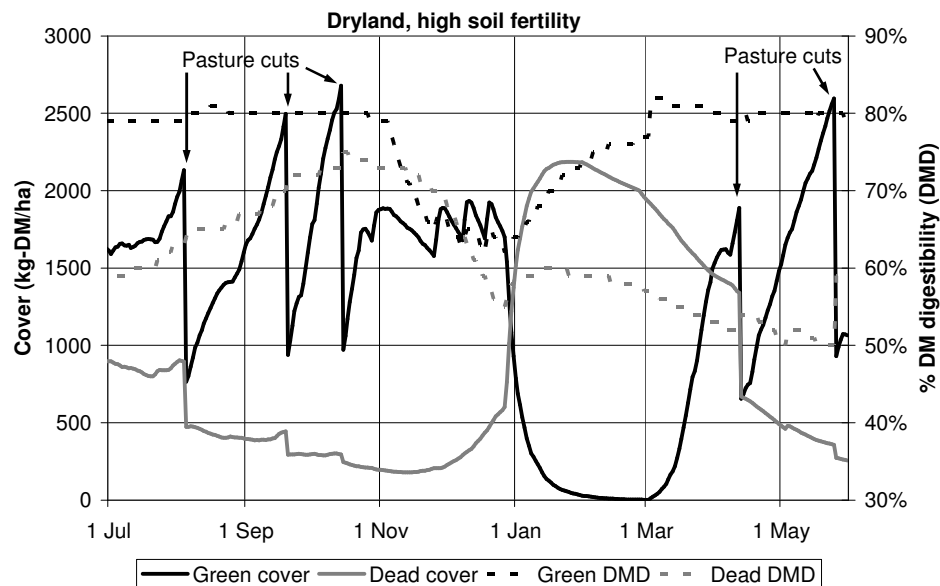
[a]



[b]



[c]



[d]

Figure 10-5: Perennial ryegrass quantity and quality (as modelled by FarmWi\$e) given Christchurch climate from July 1998-May 1999, TAW = 90 mm, a multi-layer soil, and pasture cuts when the total cover is 2,800 kg-DM/ha, cut down to 1,000 kg-DM/ha. Green material includes both the models established and senescing pools, while dead material includes the dead and litter pools.

The simulated pasture quantity estimates in Table 10-3 are reasonable for the Canterbury Plains region. For well-managed pastoral farms, typical dryland production is about 5-9 t-DM/ha/yr, and irrigated production (using high-yielding ryegrass species and spray irrigation) is about 18-20 t-DM/ha/yr (Lucock 2007). Simulated dryland pasture production (for a multi-layer soil), given average soil fertility, was 5-7 t-DM/ha/yr. Average soil fertility is more appropriate than high fertility for

dryland conditions, since the high yield ryegrass (assumed in modelling) will be out-competed by lower yielding species, which have higher drought resistance (Section 2.4.2.2). Simulated irrigated pasture production was 14-16 t-DM/ha/yr for average soil fertility, and 20-22 t-DM/ha/yr for high fertility. This compares well with observed irrigated production of up to 18-20 t-DM/ha/yr.

In Table 10-3, slightly higher average pasture quality under dryland, compared with irrigated conditions, is principally due to most dryland pasture being harvested in spring, while irrigated pasture is also harvested throughout summer when pasture quality is lower. Simulated production for a multi-layer soil compared with a single-layer soil was identical under irrigation, but 20-25% less under dryland conditions (due to greater moisture stress). In Section 2.3, simulated water use using a dual-layer soil was identical to a single-layer soil under conditions of no moisture stress, but was higher under moisture stress. These two results agree since under dryland conditions, higher water use will result in greater moisture stress.

In Figure 10-5, general features of seasonal variations in pasture quality appear to be well modelled. In practice, pasture quality reduces in summer (Fleming 2006), which is predicted in Figure 10-5 where pasture quality of green tissue is depressed from November to April. Figure 10-5 also illustrate partial death (or browning off) under dryland conditions, from January to early March, where almost all above-ground plant tissue is dead. This model prediction is commonly observed in practice.

10.6 Heuristic Performance

When using the custom heuristic method in the case study, rapid SA cooling was required due to the limited number of possible objective function evaluations (Section 10.4). Figure 10-6 plots annealing profiles, generated as part of the 1981, 1991, and 2001 case study seasons. Figure 10-6 is from when the optimiser was run on the first day of the irrigation season. The number of effective decision variables was 3. This figure shows that despite the rapid annealing cooling, the optimiser is still converging to some minima, which is significantly better than solutions found from random sampling.

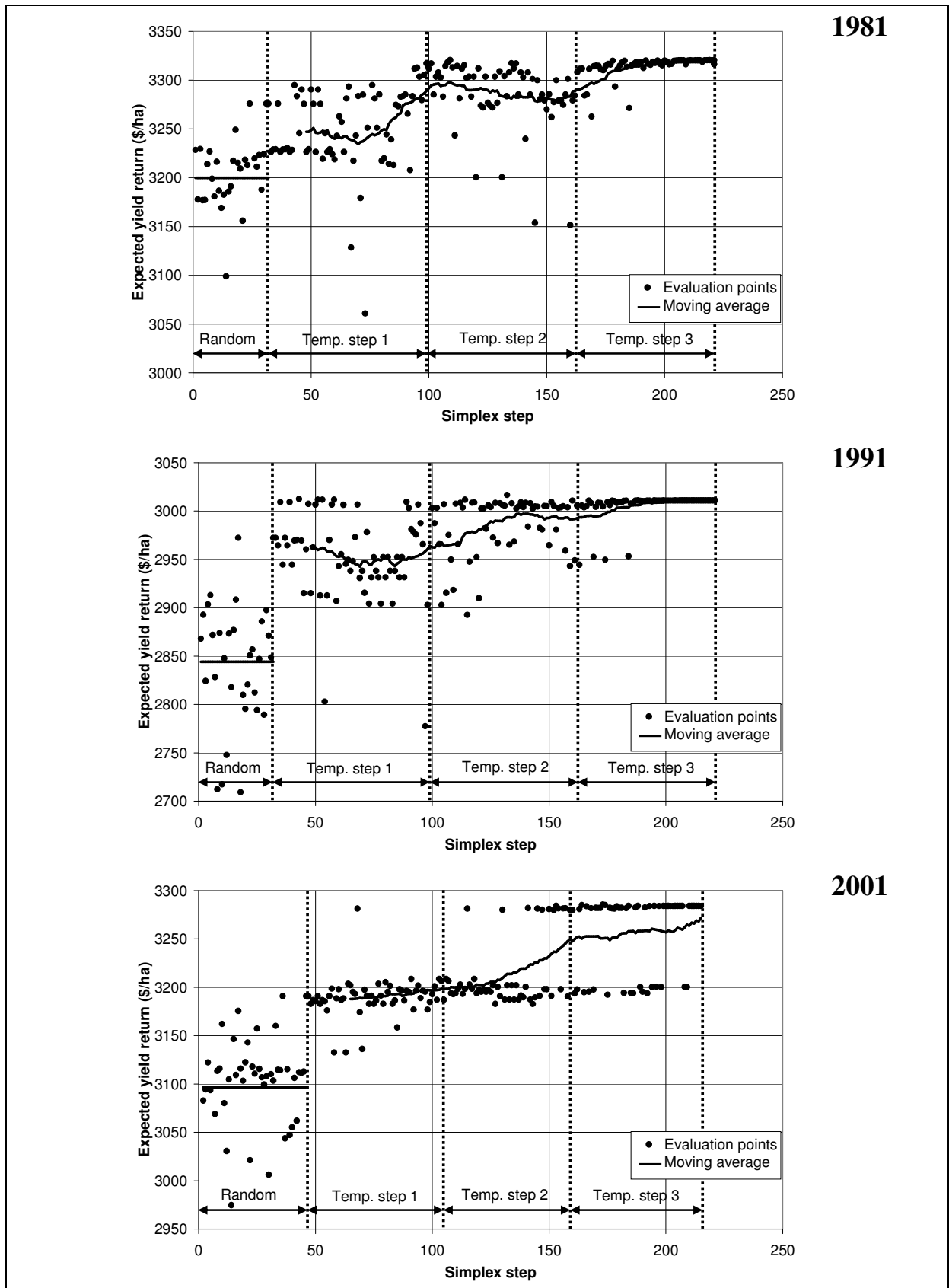


Figure 10-6: Example of SA profiles from optimisation on the first day of the irrigation season, for the case study years 1981, 1991, and 2001. A period of random sampling is followed by three temperature steps with the last step having a zero temperature.

10.7 Results

Figure 10-7 presents the annual yield returns, given the three different irrigation scheduling methods; Proposed Heuristic, Constant TSML, and best management practice²⁵. Figure 10-8 provides further details for the 1981/82 season of how different scheduling methods affected the timing and quality of individual pasture cuts.

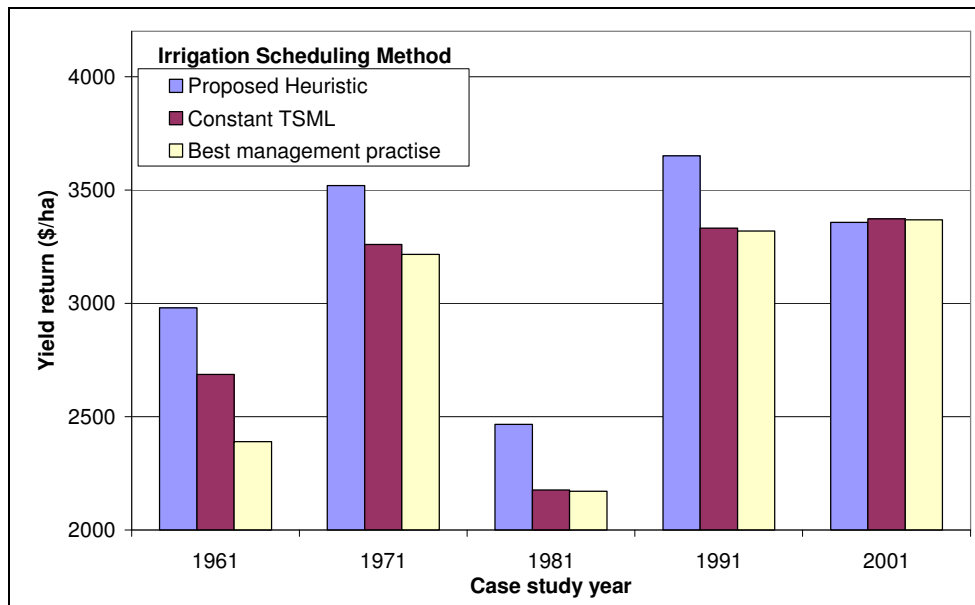


Figure 10-7: Annual yield return, as simulated by FarmWi\$, given different methods of scheduling irrigation.

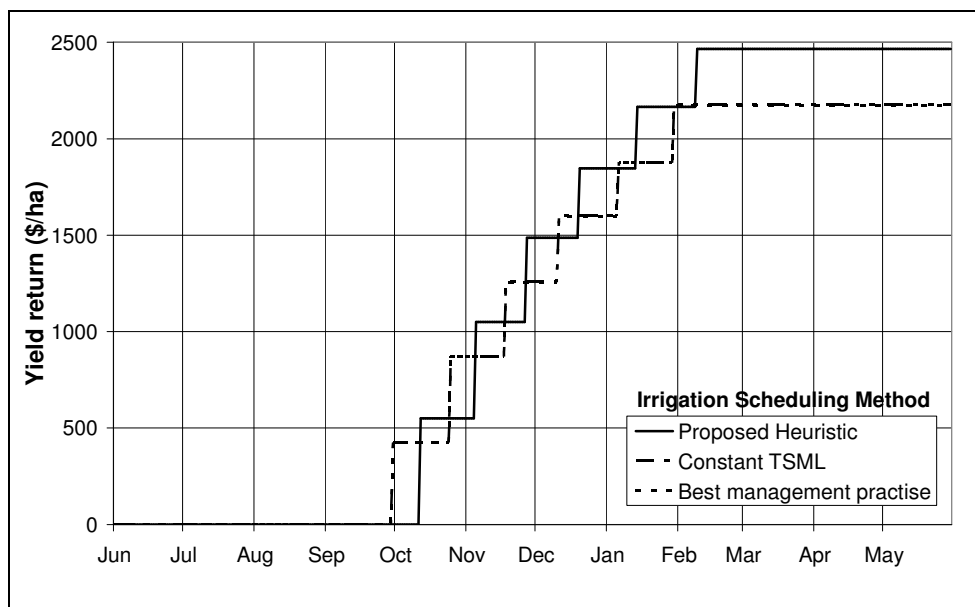


Figure 10-8: Cost function for the 1981/1982 case study season. The timing and step increases in the cost function correspond to the timing and yield value of individual pasture cuts. For this season the cost function for scheduling using the Constant TSML method and best management practice are almost identical.

²⁵ Best management practice refers to the current 'state of the art' irrigation management in New Zealand, which would typically occur when an extension service is contracted to schedule irrigation.

Figure 10-7 shows scheduling using the SA scheduler resulted in significantly greater yield returns in four out of the five case study seasons. In the fifth season, all three scheduling methods resulted in comparable returns. The lack of any advantage in scheduling using the SA scheduler for the 2001/2002 season is likely due to the seasonal water limit being in excess of that required to provide full irrigation (Section 10.3.2). The yield return averaged over the five case study seasons (relative to best management practice) was 10.4% and 2.5% higher than when scheduling with the SA scheduler and the Constant TSML method, respectively. Using the water use to yield return relationship from Figure 10-2, the SA scheduler's improvement of yield value by 10% can alternatively be expressed as a 23% reduction in water use (given no change in yield return). This is illustrated in Figure 10-9.

A paired *t*-test was used to test the null hypothesis the mean performance of the three scheduling methods were identical. The percent differences in annual yield return (compared with best management practice) for the SA scheduler and Constant TSML had a mean and standard deviation of 11.5% and 9.0%, and 2.9% and 5.3% respectively. The five case study seasons corresponded to a sample size of 5. The resulting *t* statistic was 2.8 and 1.2 for the SA scheduler and the Constant TSML method, respectively²⁶. These *t* statistics correspond to a single tail null hypothesis rejection probability of 2% and 15% respectively. Therefore, it may be concluded with some confidence that scheduling irrigation using the SA scheduler resulted in improved mean annual yield returns (compared with best management practice). Further case study seasons are required in order to conclusively demonstrate that Constant TSML method is an improvement on best management practice for this case study.

In Figure 10-8, the SA scheduler was able to schedule irrigation such, that even though the number of cuts was not increased (compared with alternative scheduling techniques); the quality of individual pasture was increased, resulting in a greater annual yield return. For this case study year, scheduling using the Constant TSML method and best management practice resulted in an almost identical cost function.

²⁶ $0.115/(0.09 \sqrt{5})=2.8$; $0.029/(0.053 \sqrt{5})=1.2$

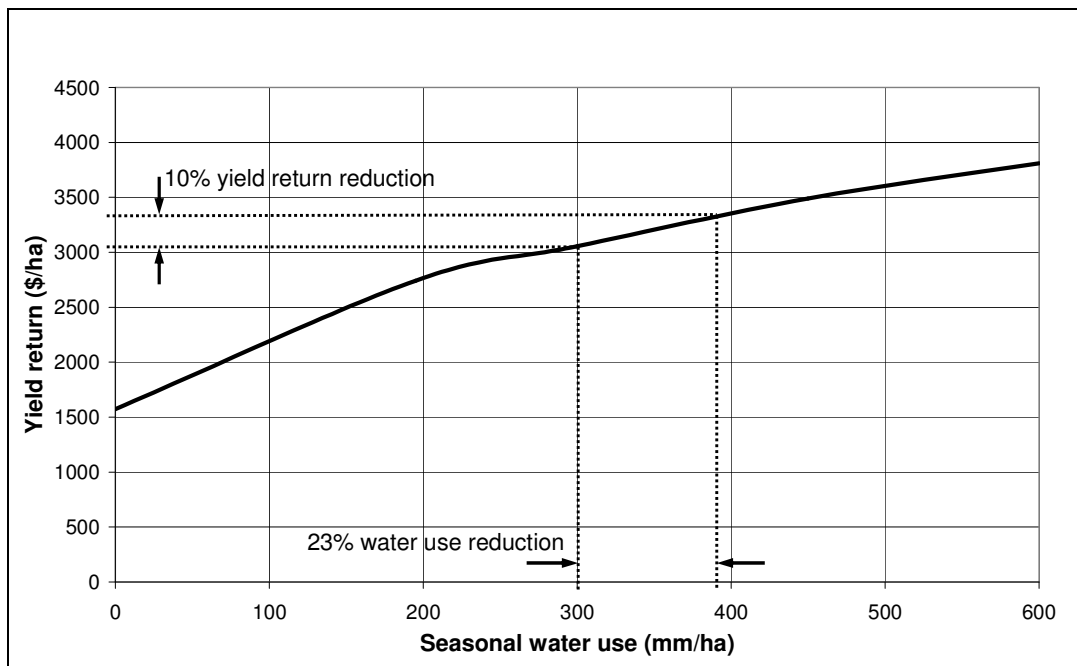


Figure 10-9: Illustrating the SA scheduler's improvement of yield value by 10% (compared with irrigation scheduling using best management practice) can alternatively be expressed as a 23% reduction in water use (given no change in yield return)

10.8 Conclusions

The chapter demonstrated the ability of the SA scheduler to incorporate complex farm system models. Preliminary comparisons showed good potential for FarmWi\$e to model well the quantity and quality of pasture production on the Canterbury Plains. A case study indicates that under conditions of limited seasonal water, the SA scheduler may be able to increase pasture yield returns by 10%, compared with scheduling irrigation using best management practice. Alternatively expressed, this corresponds to a 23% reduction in seasonal water use (given no change in the yield return).

11 CONCLUSIONS AND FUTURE RESEARCH

11.1 Conclusions

A novel optimal multi-crop irrigation scheduling algorithm (SA scheduler) was developed which was able to incorporate complex farm system models, and constraints on daily and seasonal water use, with the objective of maximising farm profit. This scheduling method included a complex farm simulation model in the objective function, used decision variables to describe general management decisions, and used a custom heuristic method for optimisation. A case study comparison with scheduling using stochastic dynamic programming demonstrated that this algorithm was able to produce close to optimal schedules. A case study incorporating the FarmWi\$e model (CSIRO, Canberra) demonstrated the possibility of incorporating complex farm system models, and the ability of the SA scheduler to increase the value of pastoral yield in Canterbury by 10%, given limits on seasonal water use.

Within the SA scheduler estimates of mean future irrigation water use, as a function of an irrigation management strategy, are required. The main factor influencing water use is climate; therefore the stochastic properties of evapotranspiration and rainfall were quantified. For evapotranspiration, using mean evapotranspiration values as a function of the time of year, was shown to be a reasonable model. For rainfall, a compound-Poisson model was shown to be acceptable. Water use is commonly estimated from the FAO 56 method (Allen et al. 1998) either with a single or dual ET crop coefficient, and with either a single or dual layer soil water balance model. A single layer soil model was shown to predict up to 15% greater water use (under deficit irrigation) compared with a dual layer soil model. Three different methods were proposed for calculating mean water use, as a function of the irrigation management strategy: (a) simulation of multiple seasons; (b) Derived probability distributions (which assume a continuous time domain); and (c) Markov chains (which assume a discrete time domain). Both the Derived probability distributions and Markov chain methods modelled evapotranspiration using mean values as a function of the time of year, and rainfall as a compound-Poisson model. The Markov chain method was shown to offer the most advantages, since (given a single soil layer and single crop coefficient model) it was 2-3 orders of magnitude faster than calculating mean water use by simulating multiple seasons, and had greater accuracy than the Derived probability distributions method. The Markov chain method had the added benefit of being able to easily model spatial variability in soil moisture.

The custom heuristic method was based on the continuous variable simulated annealing algorithm by Press et al. (2002). A comparison between continuous and discrete variable simulated annealing showed that optimising in continuous space was more efficient than optimising in discrete space. Two main extensions to Press et al.'s method were the incorporation of equality constraints and utilisation of population information. The novel method for incorporating constraints improved optimisation

efficiency, compared with traditional methods of constraint incorporation. The utilisation of population information was shown to be beneficial.

Using a case study which assumed a simple farm system model, the SA scheduler was shown to match the optimal performance of scheduling using stochastic dynamic programming. This demonstrated that the scheduler was able to produce close to optimal solutions, given sufficient computational time. Using a case study which modelled in detail the farm-system, the ability of the SA scheduler to incorporate complex farm system models was also demonstrated. The complex model used was FarmWi\$e by CSIRO Plant Industry. The advantages of this model included: (a) field verification on pastoral farms that are similar to typical NZ pastoral farms; (b) several peer-reviewed publications; (c) a track history of use by the Australian farming community as a real-time decision support tool; and (d) modern software architecture. The custom heuristic method was shown to converge to superior solutions even under conditions of rapid simulated annealing cooling. Preliminary comparisons showed good potential for FarmWi\$e to model well the quantity and quality of pasture production on the Canterbury Plains. The case study indicated that under conditions of limited seasonal water, the SA scheduler can increase the value of pasture yield by 10%, compared with scheduling irrigation using best management practice. Alternatively expressed, this corresponded to a 20-25% reduction in seasonal water use (given no change in yield value).

The benefits of utilising forecasting information were also investigated, however further work is required in this area.

11.2 Implications

Improved water use efficiency has the potential to both increase farm profit and/or decrease water use, and to reduce nitrogen leaching. Farm system models which accurately predict the effects of different farm management practices are becoming readily available. These farm models, in conjunction with the proposed optimal irrigation scheduler, are likely to be increasingly cost-effective to implement on individual farms. These tools could provide policy advice, particularly relating to how the financial impacts (on individual farmers) of seasonal water limits can be mitigated. Incorporation of the algorithm into FarmSim (by Lincoln Ventures Ltd and collaborators), which can now be run from a web-server but has yet to be fully tested, may mean that this irrigation optimisation tool will have uptake by farmers in New Zealand within the next 3-5 years.

11.3 Further research

Potential areas for future research are the further development of the Markov Chain Water Use Equation, the custom heuristic method, the field verification of FarmWi\$e for NZ farming operations, and further case study applications of the SA scheduler to a range of farm operations using complex farm models.

Further work should be undertaken to extend the Markov Chain Water Use Equation to include a dual ET crop coefficient and a dual soil layer. This equation appears promising for applications other than optimal irrigation scheduling. The main strength of the Markov chain approach is the ability to incorporate a number of stochastic variables in an efficient manner, thereby providing an alternative to Monte-Carlo Simulations. An example of a possible alternative application is quantifying how spatial variability in nitrogen application (i.e. cow urine patches) influences nitrogen leaching.

Further work into the potential benefits and limitations of Population Analysis methodology used in the custom heuristic method could be useful. One particular aspect, that was not explored, was the benefits of fixing of decision variables. However, initial results are positive and suggest that the basic concept will generally improve optimisation efficiency, particularly at higher dimensionality. The use of an adaptive cooling regime for simulated annealing also appears promising and requires further investigation.

FarmWiSe is a well engineered and flexible model, capable of modelling a wide range of NZ farming operations. No other models currently used in NZ for modelling farm systems have the breadth or accessibility of this model. A wide range of applications for this model exist in NZ outside optimal irrigation scheduling. Preliminary comparisons suggest FarmWiSe is able to accurately predict pasture quantity and quality in the Canterbury region. However more in-depth field verification of FarmWiSe for NZ farming operations and conditions is required.

A single case study was presented where the SA scheduler was applied to the farming operation of growing and cutting grass. Some simplifications were required in this case study, partly because of the further extensions required for the Markov Chain Water Use Equation, and partly because of some implementation difficulties. Further work is required to address these implementation difficulties, associated with thread termination. Future research should also extend the range of farm operations for which the SA scheduler is applied. This would allow cost benefit analysis, allowing the farming community to assess the SA scheduler.

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GLOSSARY

1) Symbols

General naming convention:

Fixed values = Capitals

Variables = Small letters

Functions = e.g. $f_{\text{TSM}}(t)$

Vectors = Bold, lower case; element reference $\mathbf{a}[i]$; Dimensions $\mathbf{a}[x] = x$ elements

Matrixes = Bold, upper case; element reference $\mathbf{A}[i,j]$; Dimensions $\mathbf{A}[x,y] = x$ rows & y columns

Irrigation system:

N_T = Number of Time Aggregation Periods (TAP) per season

N_B = Number of blocks

N_P = Number of paddocks

SC_F = Farm Portion system capacity (m^3/day or mm/day)

SC_B = Block system capacity (m^3/day or mm/day)

SC_P = Paddock system capacity (m^3/day or mm/day)

SWU = Seasonal water use limit (m^3/day or mm/day)

A_B = Area of Block (ha)

A_F = Area of Farm [portion] (ha)

D = Mean irrigation infiltration depth

$d^* = \frac{\text{Mean irrigation infiltration depth}}{\text{Soil moisture deficit}}$

UCC = Christiansen's coefficient of uniformity (Christiansen 1941)

$q = \text{Application efficiency} = \frac{\text{Applied water retained in the root zone}}{\text{Application depth}}$ (unitless)

Soil:

TAW = Total (plant) available water (mm)

RAW = (Plant) readily available water (mm)

θ = Soil moisture (mm)

θ_d = Soil moisture deficit (mm)

Plant:

k_c = Basal single crop coefficient (unitless)

k_s = Water stress reduction coefficient (unitless, 0-1.0)

Climate:

a) Evapotranspiration

et_o = Reference evapotranspiration (mm/day)

e_p = Potential evapotranspiration (mm/day) = $k_c et_o$

e_a = Actual evapotranspiration (mm/day) = $k_s e_p$

eto'_t = Standardise daily ETo at time t (unitless)

$\overline{eto'_t}$ = Mean standardise daily ETo at time t (unitless)

b) Precipitation

u = Precipitation infiltration depth (mm)

λ = Mean number of storm events per day

S = Total number of storms

W = Total number of wet days

T = Total number of days

\bar{u}_s = Mean infiltrated storm depth (mm)

u_w = Infiltrated wet day precipitation depth (mm)

\bar{u}_w = Mean infiltrated wet day precipitation depth (mm)

σ_s^2 = Variance of storm infiltration depths (mm²)

σ_w^2 = Variance of wet day precipitation depths (mm²)

$\sigma_s = \frac{\bar{u}_s}{\sigma_s^2} = \text{Gamma scale parameter for infiltrated storm depths (mm}^{-1}\text{)}$

$\beta_s = \frac{\bar{u}_s^2}{\sigma_s^2} = \text{Gamma shape parameter for infiltrated storm depths (unitless)}$

$PD_s(n)$ = Probability distribution of n events occurring within one day

PDF_u = Probability density function of infiltrated storm depths u (mm)

PDF_{u_w} = Probability density function of infiltrated wet day rain depths (mm)

Derived probability distribution water use equation:

u_e = Effective precipitation (mm)

q = a = Proportion of infiltrated precipitation retained within the root zone

r = Mean daily infiltrated precipitation (mm/day)

e_e = Mean effective evapotranspiration

θ_{\max} = Maximum soil moisture level (given Derived Probability Distribution assumptions)

θ_{\min} = Minimum soil moisture level (given Derived Probability Distribution assumptions)

Markov Chain Water Use Equation:

I = Identity matrix (unitless)

θ_{pv} = Soil moisture probability vector (unitless)

θ_{top} = Soil moisture state discretisation vector – top of discretisation intervals (mm)

θ_{mid} = Soil moisture state discretisation vector – mid-point of discretisation intervals (mm)

θ_{btm} = Soil moisture state discretisation vector - bottom of discretisation intervals (mm)

- Θ = Transformation matrix for soil moisture rediscratisation (unitless)
- E_a = Actual evapotranspiration probability transition matrix (unitless)
- U_e = Effective precipitation probability transition matrix (unitless)
- I_g = Irrigation probability transition matrix (unitless)
- i_g = Irrigation depth vector (mm)

2) Mathematical functions

$\text{Exp}(z)$ = Exponential function

$$\text{Erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z t^{z-1} e^{-t} dt = \text{Error function}$$

$\text{Erfc}(z) = 1 - \text{Erf}(z)$ = Complementary error function

$$\text{GD}(z|x,y) = \frac{(x^y e^{-xz}) z^{y-1}}{\Gamma(y)} = \text{Gamma distribution with parameters } x \text{ and } y \text{ and variable } z$$

$\ln(z)$ = Natural logarithm of z

$$\Gamma(z) = \int_0^\infty t^{z-1} \text{Exp}(-t) dt = \text{Euler's Gamma function}$$

$$\Gamma(a,z) = \int_z^\infty t^{a-1} \text{Exp}(-t) dt = \text{Incomplete Gamma function}$$

3) Textual abbreviations

- AR = Auto regression
- ARMA = Auto-regression moving average
- DP = Dynamic programming
- DPD = Derived Probability Distribution
- CMP = Common Modelling Protocol
- ET = Evapotranspiration
- ETo = Reference evapotranspiration
- FAO = (United Nations) Food and Agricultural Organisation
- FAW = Fraction of (plant) available water
- GA = Genetic algorithm
- LP = Linear programming
- NZ = New Zealand
- PDF = Probability density function
- PTM = Probability transition matrix
- PV = Probability vector
- RAM = Random access memory
- RAW = Readily [plant] available water
- SA = Simulated annealing
- SDP = Stochastic dynamic programming

SOI	= Southern Oscillation Index
TAP	= Time aggregation period
TAW	= Total (plant) available water
TE	= Truncation error
TSML	= Trigger soil moisture level
UCC	= Christiansen's coefficient of uniformity (Christiansen 1941)

4) Defined proper names

Irrigation unit	= Single physical irrigation unit (Table 2.2)
Irrigation system	= One or more identical irrigation units dedicated to a specified region of the farm (Table 2.2)
Paddock	= Area that can be irrigated in one day by a single irrigation unit (Table 2.2)
Block	= Collection of Paddocks that have homogeneous crop and soil properties, serviced by one or more identical irrigation units (Table 2.2)
Portion	= Collection of Blocks that share the same Irrigation System (Table 2.2)
SA scheduler	= The authors optimal irrigation scheduler which uses simulated annealing for optimisation (Chapter 6)
Farm Irrigation Strategy	= Collection of Irrigation Trigger Soil Moisture Levels (as a function of time, from the current day to the end of the irrigation season) for each Block within a farm Portion (Section 6.4)
Block Irrigation Strategy	= Collection of Irrigation Trigger Soil Moisture Levels (as a function of time, from the current day to the end of the irrigation season) for a single Block (Section 6.4)
Water Use Equation	= Equation relating a irrigation regime over a period of time, to the mean water use (Chapter 7)

5) Unit abbreviations

l	= Litre
m	= Metre
ha	= Hectare
s	= Second
d	= Day (also dy)
yr	= Year
ms	= Milli-seconds
MJ	= MegaJoule
GHz	= Gigahertz
MB	= Mega-Byte
GB	= Giga-Byte

APPENDIX 1

Electronic copy of source code and input data